GRID-BASED AUTOMATIC 3D MESH GENERATION FROM THE PLANAR CROSS-SECTIONS

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ABSTRACT

From a series of planar cross-sections, a new tetrahedral meshing algorithm using a tetrahedral grid template is proposed with several assumptions. In this scheme, the diagonal projection of the planar section on the above plane onto the bottom plane is considered. It is shown that our method basically depends only on a simple projection of a plane in 3D and the mesh generation method in 2D based on a equilateral grid-based template.

INTRODUCTION

In medical field, many three dimensional objects are visualized by constructing surface meshes from the 2D scalar-valued slices obtained through computed tomography (CT), magnetic resonance image (MRI) and ultrasound imaging. For the purpose of finite element analysis, it is necessary to construct tetrahedra for the solid bounded by the surface meshes. However, although much research efforts have been made on surface construction from contours, relatively few works have been made on constructing three dimensional finite element models from these contours. Also, most of their works first require triangular surface mesh construction from the slice data, and secondly conduct tetrahedral element mesh generation of the solid domain bounded by the surface mesh by using the methods such as an advancing front algorithm.

In this paper the diagonal projection of the planar section on the above plane onto the bottom plane is considered. Then two domains with cross-sections as their boundaries are divided into two types : non-overlapped parts belonging to only one cross section (above or bottom section) and overlapped parts. Then the tetrahedral meshes of overlapped parts are automatically constructed using automatic 2D mesh generation based on an equilateral grid-based template. For non-overlapped parts, the method of making tetrahedral meshes is proposed. Therefore, we obtain tetrahedral meshes without using surface construction from contours unlike other works. In addition, the surface construction is automatically obtained after making tetrahedral meshes.

REVIEW OF TWO DIMENSIONAL MESH GENERATION BASED ON GRID-TEMPLATE

Assumption

$\mathcal{D}$ is a multiply connected bounded polygonal domain to be discretized and the boundary Γ of $\mathcal{D}$ consists of the simple closed polygons $\Gamma_1, \Gamma_2, \cdots, \Gamma_n$, where Γ_1 is positively oriented and the other Γ_j's are negatively oriented.

Let $\mathbf{p} = \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}$ be an array of the x-, y-coordinates for given points on Γ which are points on the polygon Γ_k with the given order. Let h be the average value of the lengths of all edges of Γ.

We assume that the points in **p** are quasi-uniformly distributed with the following properties.

(A4) There are no cases where two more boundary points are included in a regular hexagon with the length of one edge as the value $\frac{h}{\sqrt{3}}$.

(A5) The distance between two consecutive boundary points is always less than $2h$.

Under these settings, the purpose of this section is to recall the generating method of a good initial mesh triangulating the interior of the domain $\mathcal{D}$ with as many equilateral triangles as possible by using a grid-based template, which is mentioned at the papers [1] and [2].

Algorithm

The main process of the mesh generation algorithm can be illustrated as follows ([1], [2]):

Step 1. The equilateral mesh template is superimposed on the domain.

Step 2. Find the closest grid points to the array of the given boundary points, which compose of the pseudo-boundary. If two boundary points are corresponded to the same grid points, then the later point in order, say a non-matched point, is excluded in advance.

Step 3. Conduct two types, $C1$ and $C2$, of modifying operations.

Step 4. Find the zigzag boundary, the set of the interior points consisting of triangles with one or two boundary points.

Step 5. Find the grid cells near the zigzag boundary.

Step 6. Adjust the boundary points to the corresponding grid points.

Step 7. Add and connect the non-matched boundary points again according to the first given order.

Step 8. Conduct smoothing processes using Laplacian smoothing and remesh by aspect ratios to improve the poorly-shaped interior grid cells near to the boundary.

Step 9. Keep the interior point.

Step 9 can be practiced after step 5 as the previous section, but it is placed at the last since step 6, 7, and step 8 are executed with only grid cells near the zigzag boundary.

AUTOMATIC MESH GENERATION IN 3D

Assumption

Let $\mathbf{p} = \begin{bmatrix} \mathbf{x}\ \mathbf{y}\ \mathbf{z} \end{bmatrix}'$ be an array of the x-, y-, z-coordinates for given points on each cross-section Γ which are points on the polygon Γ_k with the given order. Let h be the average value of the lengths of all edges of all contours on planar cross-sections.

(A1) If the distance between two adjacent cross- sections has the value of l, then the mesh size h of tetrahedrons has approximately the same value of $\frac{\sqrt{6}}{2}\,l$.

(A2) There are no cases where two more boundary points are included in a regular hexagon with the length of one edge as the value $\dfrac{h}{\sqrt{3}}$.

(A3) The distance between two consecutive boundary points is always less than $2h$.

Algorithm

The basic idea is as follows :

(1) Make a volume grid-template with two planar cross-sections as shown in Fig. 1(a).

(2) As shown in Fig. 1(b), we project diagonally the top facet onto the bottom facet to correspond the grid points of the top facet with the grid points of the bottom facet. Then the algorithm divides the domain into two types: (a) the non-overlapped part belonging to the top facet or the bottom facet and (b) the overlapped part.

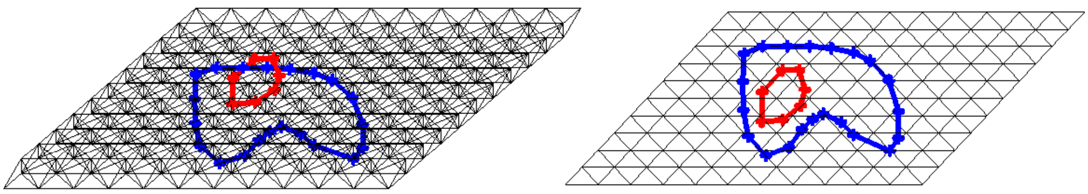

Figure 1. (a) 3D grid template (b) Diagonal projection

After diagonal projection on bottom template of the above planar cross-section, the main algorithm is as follows:

Step 1. The overlapped domains are triangulated on the bottom facet using the 2D mesh generation algorithm introduced by the previous section and are diagonally pulled up back onto the top facet using the method of Fig. 2 (see Fig. 3).

Step 2. Triangulate non-overlapped parts using the 2D mesh generation. Then tetrahedral meshes at non-overlapped parts are made with triangles on their own cross-section and the wall

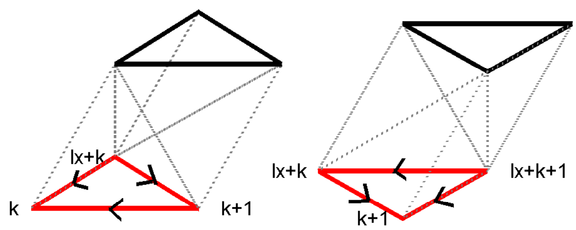

Figure 2. Tetrahedrons on a unit of 3D mesh

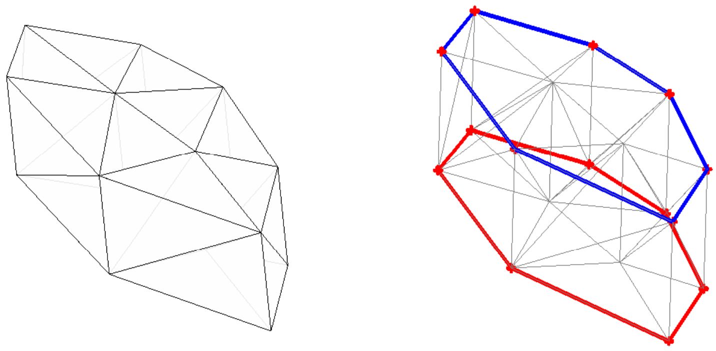

Figure 3. Tetrahedrization for overlapped part

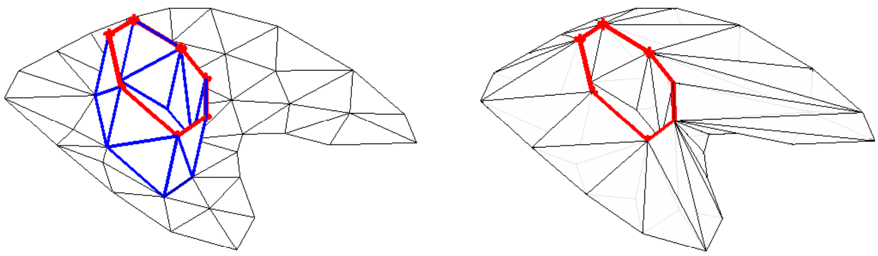

Figure 4. Tetrahedrization for non-overlapped part

obtained from step 2. (see Fig. 4)

REFERENCES

1. Kim, P. and Ahn, S., “Automatic initial mehs generation by a grid-based template”, *Proc. of KSIAM 2006 Spring Conference*, 2006
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