Positive scalar curvature metric on submanifolds of quasi-toric manifolds

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Positive scalar curvature metric

1. Introduction of positive scalar curvature metric on manifolds
2. Positive scalar curvature metric on quasi-toric manifolds
3. α invariant for submanifolds of quasi-toric manifolds

Curvature

For a Riemann manifold (M^n, g), R be the curvature tensor.

Sectional curvature

Let $E \subset T_p(M)$ be a two-dimensional subspace, X, Y be two linear independent vectors in E, then

$$K_p(E) = R(X, Y, X, Y)/(\|X\|^2\|Y\|^2 - \langle X, Y\rangle^2)$$

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Ricci curvature

For fixed $X, Y \in T_p(M)$, map $Z \mapsto R(Z, X)Y$ induces a linear transformation $T : T_p(M) \to T_p(M)$.
The trace of T is called Ricci tensor S of (M^n, g);
$Ric(X) \triangleq S(X, X)$ is called Ricci curvature along X at $p \in M$.

Scalar curvature

Ricci tensor S induces a linear transformation $R^* : T_p(M) \longrightarrow T_p(M)$ where $\langle R^*(X), Y \rangle = S(X, Y)$ for arbitary $Y \in T_p(M)$.

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Remark

If we choose a locally orthonormal tangent frame $\{e_1, \ldots, e_n\}$ at p, $Ric(X) = \sum_i R(e_i, X, e_i, X)$; $\rho = \sum_i Ric(e_i)$.
This implies Ricci curvature is the average of sectional curvature and scalar curvature is the average of Ricci curvature.

Study on positive scalar curvature metric on manifolds

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Theorem(Gromov-lawson, 1980; Schoen-Yau, 1979)

Let M be a manifold which carries a metric of positive scalar curvature, then any manifold obtained from M by surgery in codimension ≥ 3, also carries a metric of positive scalar curvature.

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Theorem (Gromov-Lawson,1980, Stolz, 1992)

Let M be a simply connected, closed, spin manifold of dimension ≥ 5. Then M carries a metric with positive scalar curvature if and only if $\alpha(M) = 0$.

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- $\alpha(M^n)$ equals to $\widehat{A}(M^n)$ up to a multiple when $n \equiv 0 \mod 4$
- Let K be a *spin*^{c} manifold with dimension $8k+4$. There exists a complex line bundle ξ, such that $c_1(\xi) \equiv \omega_2(K) \mod 2$. Let B be the oriented submanifold *Poincaré* dual to $c_1(\xi)$. Then B is a spin manifold. Weiping Zhang proved a Rokhlin type congruence fomula

$$\alpha(B) \equiv \langle \widehat{A}(K)\exp^{c_1/2}, [K] \rangle \mod 2$$

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- when $n = 2$, every 4-dimentional quasitoric manifold admits an invariant metric of positive Ricci curvature.
- when $n \geq 3$, using equivariant surgery, every moment-angle manifold admits an invariant metric of positive scalar curvature. By Berard and Bergery's result, the quotient space M^{2n} admits an invariant metric of positive scalar curvature.

□

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- when $2n \equiv 6 \mod 8$, $\alpha(M) = 0$. Since $KO_{8k+6} = 0$.

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- when $2n \equiv 6 \mod 8$, $\alpha(M) = 0$. Since $KO_{8k+6} = 0$.
- when $2n \equiv 2 \mod 8$, M is the characteristic submanifold of $K = M \times \mathbb{C}P^1$ corresponding to generator $t \in H^2(\mathbb{C}P^1)$. Using Zhang's formula,

$$\alpha(M) \equiv \langle \widehat{A}(K) \exp^{t/2}, [K] \rangle \equiv \langle \widehat{A}(M)\widehat{A}(\mathbb{C}P^1) \exp^{t/2}, [M] \times [\mathbb{C}P^1] \rangle$$

$$\equiv \langle \widehat{A}(M), [M] \rangle \langle \widehat{A}(\mathbb{C}P^1) e^{t/2}, [\mathbb{C}P^1] \rangle \equiv 0 \mod 2$$

α invariant for submanifolds of quasi-toric manifolds

Recall

Let K be a *spin*^{c} compact manifold with dimension $8k+4$.
By definition of *spin*^{c} manifolds, there exists a complex line bundle ξ, such that $c_1(\xi) \equiv \omega_2(K) \mod 2$. B is the $8k+2$-dim oriented submanifold *Poincaré* dual to $c_1(\xi)$. Then B is a spin manifold.

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Theorem(Kato, Matsumoto, 1972)

1. For any oriented manifold M, any homology class in $H_{m-2}(M;\mathbb{Z})$ is represented by a taut $K^{m-2} \subset M^m$.
2. If K is taut in M^{2n}, then the pair (M, K) is $n-1$ connected.

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Assume $H^*(K) \cong \mathbb{Z}(v_1, \ldots, v_m)/(I+J)$, where v_i corresponds to a facet F_i of P.
$T(K) \oplus \underline{\mathbb{R}}^{2(m-n)} \cong \xi_1 \oplus \cdots \oplus \xi_m$ where $c_1(\xi_i) = v_i$.
For an element $u \in H^2(K)$, $u = \sum_{i=1}^m d_i v_i$, we choose appropriate d_i such that $u \equiv \omega_2(K) \mod 2$. where $\omega_2(K) = \sum_{i=1}^m v_i$.

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Proposition

Let B be the simply connected, spin submanifold Poincaré dual to u,

$$\alpha(B)\equiv\langle\prod_{i=1}^m\frac{v_i/2}{sinh(v_i/2)}\exp^{\sum_{i=1}^m d_iv_i/2},[K]\rangle\mod 2$$

$\mathbb{C}P^{4k+2}$(Weiping Zhang)

Let K be $\mathbb{C}P^{4k+2}, \quad k > 0$. For arbitary positive integer d, we denote $V^{4k+1}(d)$ to be d degree regular hypersurface of $\mathbb{C}P^{4k+2}$.

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Propsition

If d is odd, $V^{4k+1}(d)$ is the simply connected, spin submanifold *Poincaré* dual to u.

$$\alpha(V^{4k+1}(d) \equiv \begin{cases} 0, & d \leq 4k+1 \\ C^{4k+2}_{[\frac{d}{2}]+2k+1}, & d \geq 4k+3 \end{cases} \quad \mod 2$$

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- For $\mathbb{C}P^6$, $V^{4k+1}(7), V^{4k+1}(9), \ldots$ doesn't admit positive scalar curvature metrics;

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- For $\mathbb{C}P^{4k+2}$, if d is even, $V^{4k+1}(d)$ admits a positive scalar curvature metric.
- For $\mathbb{C}P^{6}$, $V^{4k+1}(7), V^{4k+1}(9), \ldots$ doesn't admit positive scalar curvature metrics; $V^{4k+1}(11), V^{4k+1}(13), \ldots$ admit positive scalar curvature metrics.

$\prod_{i=0}^{s} \mathbb{C}P^{t_i}$

Let K be $\prod_{i=0}^{s} \mathbb{C}P^{t_i}$ such that $\sum_{i=1}^{s} t_i \equiv 2 \mod 4$. Choose an element $u \in H^2(K)$, $u = \sum_{i=1}^{s} d_i x_i$ where x_i is generator of $H^2(\mathbb{C}P^{t_i})$.

If $d_i + t_i$ is odd for any i, Let B be the simply connected, spin submanifold *Poincaré* dual to u.

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If $d_i + t_i$ is odd for any i, Let B be the simply connected, spin submanifold *Poincaré* dual to u.

$$\alpha(B) \equiv \prod_{i=1}^{s} \langle (\frac{x_i/2}{sinh(x_i/2)})^{t_i+1} \exp^{d_i x_i/2}, [\mathbb{C}P^{t_i}] \rangle \mod 2$$

We denote a_i to be $\langle (\frac{x_i/2}{sinh(x_i/2)})^{t_i+1} \exp^{d_i x_i/2}, \mathbb{C}P^{t_i} \rangle$.

Proposition

$$a_i = \begin{cases} 0 & d_i \leq t_i - 1 \\ C^{t_i}_{\frac{d_i+t_i-1}{2}} & d_i \geq t_i + 1 \end{cases}$$ *regardless d_i is odd or even.*

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Theorem

Let B *be the simply connected, spin submanifold defined above,* B *doesn't admits a positive scalar curvature metric* $\Longleftrightarrow$ *all* a_i *are odd.*

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Example

For $\mathbb{C}P^1 \times \mathbb{C}P^5$, if $d_1 = 2, 6, \ldots,$ $d_2 = 6, 10, \ldots$, B doesn't admit positive scalar curvature metrics.

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For $\mathbb{C}P^2 \times \mathbb{C}P^4$, if $d_1 = 3, 5, \ldots,$ $d_2 = 5, 7, \ldots$, B doesn't admit positive scalar curvature metrics.

Generalised Bott manifolds

Theorem

Let $p_k : B_k \longrightarrow B_{k-1}$ be the projectivisation of $\xi^{j_k} \oplus \underline{\mathbb{C}}$ over B_{k-1}, and let η be the tautological line bundle over B_k, let $v \in H^2(B_k)$ be the first Chern class of η. Then

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- $\mathcal{T}B_k \oplus \underline{\mathbb{C}} \cong p_k^*\mathcal{T}B_{k-1} \oplus (\bar{\eta} \otimes p^*(\xi^{j_k} \oplus \underline{\mathbb{C}}))$ where $\underline{\mathbb{C}}$ denotes a trivial line bundle over B_k.

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- $H^*(B_k) \cong H^*(B_{k-1})[v]/I$ where I is an idea generated by $v^{j_k+1} - c_1(\xi^{j_k})v^{j_k} + \cdots + (-1)^{j_k} c_{j_k}(\xi^{j_k})v$

We consider generalised Bott tower of height 2. $p_2 : B_2 \longrightarrow \mathbb{C}P^{j_1}$.
Let $K = B_2$ satisfying $j_1 + j_2 \equiv 2 \mod 4$,

We denote $b_i t$ to be Chern root for $\xi^{j_2} \oplus \underline{\mathbb{C}}$.
$H^*(B_2) \cong \mathbb{Z}[t, v]/I$ where I is an idea generated by
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Choose an element $u \in H^2(K), u = d_1 t + d_2 v$.

Proposition

If $d_2 + j_2$ is odd, and $j_1 + d_1 + c_1(\xi^{j_2})$ is odd, let B be the simply connected, spin submanifold Poincaré dual to u,

$$\alpha(B) \equiv \langle (\frac{\frac{t}{2}}{sinh(\frac{t}{2})})^{j_1+1} \prod_{i=1}^{j_2+1} \frac{\frac{v+b_i t}{2}}{sinh(\frac{v+b_i t}{2})} \exp^{(\frac{d_1 t + d_2 v}{2})}, [B_2] \rangle \mod 2$$

$$\alpha(B) \equiv \sum_{k=0}^{j_1} A_k B_k \mod 2$$

Where A_k denotes the coefficient of $t^{j_1-k}v^{j_2+k}$ in polynomial $(\frac{\frac{t}{2}}{sinh(\frac{t}{2})})^{j_1+1}\prod_{i=1}^{j_2+1}\frac{\frac{v+b_i t}{2}}{sinh(\frac{v+b_i t}{2})}\exp^{(\frac{d_1 t+d_2 v}{2})}$, and B_k denote $\langle t^{j_1-k}v^{j_2+k}, [B_2]\rangle$.

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Proposition

- $B_k =$ $$\sum_{\alpha_1+2\alpha_2+\cdots+j_2\alpha_{j_2}=k} (-1)^{k+\alpha_1+\ldots\alpha_{j_2}}\frac{(\alpha_1+\ldots\alpha_{j_2})!}{\alpha_1!\ldots\alpha_{j_2}!}c_1^{\alpha_1}\ldots c_{j_2}^{\alpha_{j_2}}$$
- $A_k =?$

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