The Complexity of Register Allocation

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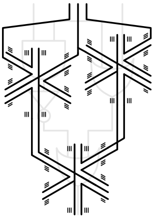

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Register Allocation

- Programs use variables that have to be stored somewhere
- Compiler decides where to store variables
  - Registers vs. memory
  - Very important for code quality

C Code

```
int binomial_coefficient(int n, int k)
{
        int i, delta, max, c;
        if(n < k)
                return(0);
        if(n == k)
                return(1);
        if(k < n - k)
        {
                delta = n - k;
                max = k;
        }
        else
        {
                delta = k;
                max = n - k;
        }
        c = delta + 1;
        for(i = 2; i <= max; i++)
                c = (c * (delta + i)) / i;
        return(c);
}
```

Control Flow Graph

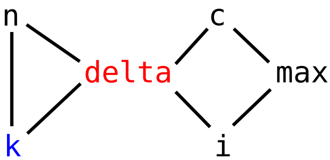
```
int binomial_coefficient(int n, int k)
{
        int i, delta, max, c;
        if(n < k)
                return(0);
        if(n == k)
                return(1);
        if(k < n - k)
        {
                delta = n - k;
                max = k;
        }
        else
        {
                delta = k;
                max = n - k;
        }
        c = delta + 1;
        for(i = 2; i <= max; i++)
                c = (c * (delta + i)) / i;
        return(c);
}
```

Live Range

```
int binomial_coefficient(int n, int k)
{
        int i, delta, max, c;
        if(n < k)
                return(0);
        if(n == k)
                return(1);
        if(k < n - k)
        {
                delta = n - k;
                max = k;
        }
        else
        {
                delta = k;
                max = n - k;
        }
        c = delta + 1;
        for(i = 2; i <= max; i++)
                c = (c * (delta + i)) / i;
        return(c);
}
```

Conflict Graph

```
int binomial_coefficient(int n, int k)
{
        int i, delta, max, c;
        if(n < k)
                return(0);
        if(n == k)
                return(1);
        if(k < n - k)
        {
                delta = n - k;
                max = k;
        }
        else
        {
                delta = k;
                max = n - k;
        }
        c = delta + 1;
        for(i = 2; i <= max; i++)
                c = (c * (delta + i)) / i;
        return(c);
}
```



Register Allocation

Given an input program and parameter r, the number of registers and a number g, the problem of *register allocation* is to decide if there is an r-colorable induced subgraph S in the conflict graph, such that the sum $\sum_{v \in V \setminus V(S)} c(v)$ of the costs of the variables outside this subgraph is at most g.

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Graph Coloring Register Allocation[1]

Assuming all r registers are equal:

- Registers are colors.
- Color the conflict graph.
- Placing as many variables in registers as possible is equivalent to finding a maximum r-colorable induced subgraph.

[1] G. Chaitin, 1982

Problems with Graph Coloring

- NP-hard:
  Approximations are used, leading to suboptimal colorings.
- Hard to generalize:
  Register preferences, aliasing, etc are hard to model.
- Solution to NP-Hardness: Restricting the class of input graphs.

Tree Width

- Many problems that are NP-hard can be solved efficiently when restricted to graphs of bounded tree-width.
- The tree-width of control-flow graphs is bounded[2].
- For structured programs, and fixed *r* the decision problem of *r*-colorability of the conflict graph can be solved in linear time[3].
- For structured programs, and fixed *r* the register allocation problem can be solved in polynomial time[4].

[2] M. Thorup, 1998

[3] H. Bodlaender et alii, 1998

[4] K.

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Weighted Circuit Satisfiability

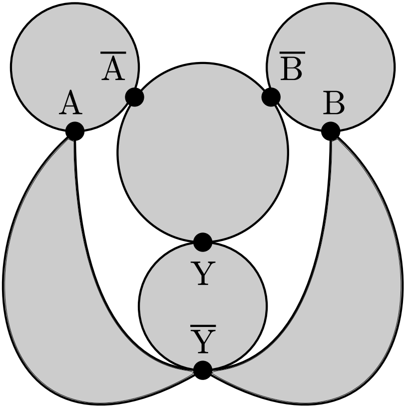
Given a circuit C (input) and an integer k (parameter) the weighted circuit satisfiability problem asks if there is a satisfying assignment for C of weight exactly k (i. e. exactly k of the inputs are set to "true" in the assignment).

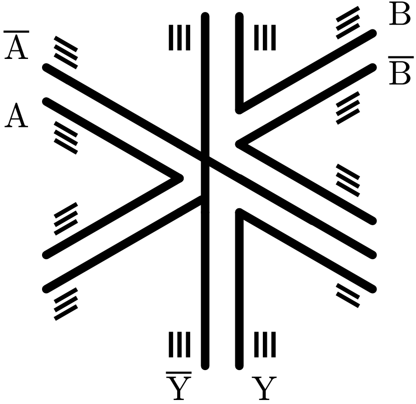
Parametrized Complexity Classes

- fpt is the class of all parametrized problems parametrized by k that can be solved in time $f(k)p(n)$ for input size n, computable f and polynomial p.
- $W[t]$ is the class of all parametrized problems that can be fpt-reduced to the WCS problem on weft t, depth d SAT-circuits, for a constant $d \geq 1$.
- $W[\mathrm{SAT}]$ is the class of all parametrized problems that are fixed-parameter reducible to the WCS problem on SAT-circuits.
- $W[P]$ is the class of all parametrized problems that are fixed-parameter reducible to the WCS problem.
- XP is the class of all problems parametrized by k that can be solved in time $f(k, n)$, with n being the size of the input and f polynomial in n for fixed k.

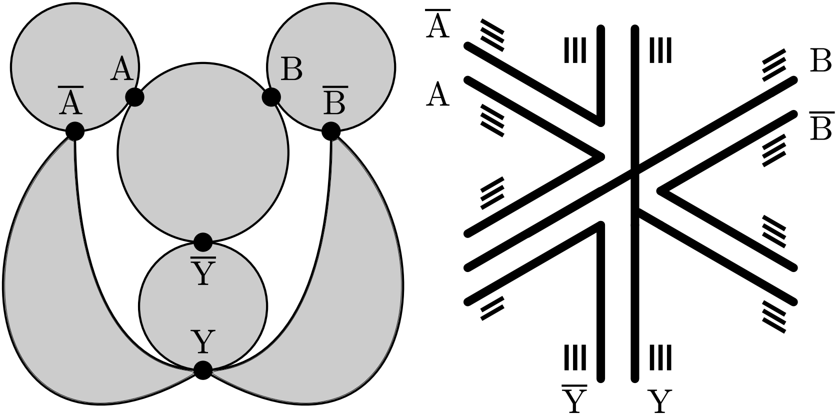
$$\mathrm{fpt} \subseteq W[1] \subseteq W[2] \subseteq \ldots \subseteq W[\mathrm{SAT}] \subseteq W[P] \subseteq XP.$$

OR


A
$\overline{\mathrm{A}}$
$\overline{\mathrm{B}}$
B
Y
$\overline{\mathrm{Y}}$

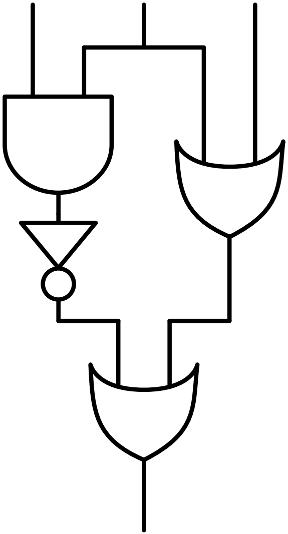

$\overline{\mathrm{A}}$
A
B
$\overline{\mathrm{B}}$
$\overline{\mathrm{Y}}$
Y

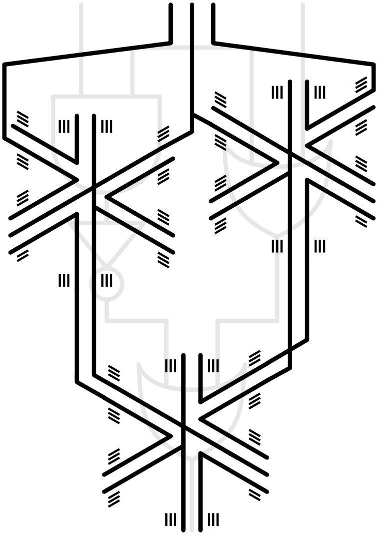
AND



A
B
$\overline{\mathrm{A}}$
$\overline{\mathrm{B}}$
$\overline{\mathrm{Y}}$
Y
$\overline{\mathrm{A}}$
A
B
$\overline{\mathrm{B}}$
$\overline{\mathrm{Y}}$
Y

Example transformation





WCS reduced to register allocation

- The tree-width of the control-flow graph is at most one more than the tree-width of the circuit.
- The node where all the live ranges of inputs meet ensures that at most $r = k$ of the inputs are assigned the value "true".
- Adjusting the weights we can ensure that finding an optimal assignment of variables to registers results in a "yes" or "no" answer for the WCS problem.

Hardness of register allocation

The register allocation problem, when parametrized by the number of registers r is $W[\mathrm{SAT}]$-hard, even for structured programs of tree-width 2.