Partitions of Chordal Graphs

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Co-authors

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Graph Partitions

Graph Partitions

Parts and their connections are constrained

Assign vertices to parts so that constraints are satisfied

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Assign vertices to parts so that constraints are satisfied

Constraints on parts

- Independent set
- Clique
- Unrestricted

Graph Partitions

Parts and their connections are constrained

Assign vertices to parts so that constraints are satisfied

Constraints between parts

- No edges
- All edges
- Unrestricted

Graph Partitions

Parts and their connections are constrained

Assign vertices to parts so that constraints are satisfied

Partition into k parts

A k by k symmetric matrix M over $0, 1, *$

- No edges = 0
- All edges = 1
- Unrestricted = *

Graph Partition

A k by k symmetric matrix M over $0, 1, *$

An M-partition of G

Graph Partition

A k by k symmetric matrix M over $0, 1, *$

An M-partition of G

A partition $V_1, V_2, \ldots, V_k$ of $V(G)$ with

- V_i a clique if $M(i,i) = 1$, and independent set if $M(i,i) = 0$
- all edges between V_i and V_j present if $M(i,j) = 1$, and absent if $M(i,j) = 0$

Graph Partition

A k by k symmetric matrix M over $0, 1, *$

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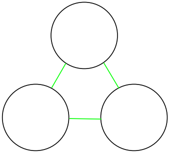
The M-partition problem

Decide if input graph G has an M-partition

Example

Matrix M

$$\begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$



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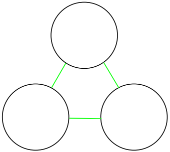

The M-partition problem

Is G 3-colourable?

NP-complete

Example

Matrix M

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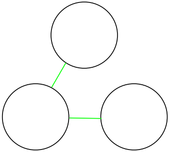

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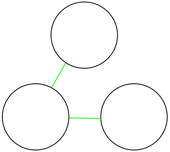


The M-partition problem

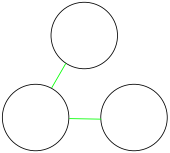
Does G admit a homomorphism to P_3?

Polynomial time solvable

Example

Matrix M

$$\begin{pmatrix} 0 & * & * \\ * & 0 & 0 \\ * & 0 & 0 \end{pmatrix}$$



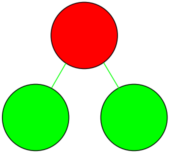
M-partition problems

Generalize graph colouring and homomorphism problems

Example

Matrix M

$$\begin{pmatrix} 1 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix}$$



Example

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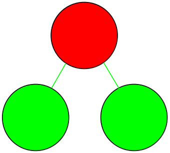

An M-partition of G (with all parts nonempty)

Does G have a clique cutset?

Polynomial time solvable (Whitesides 1981, Tarjan 1985)

Variants of M-partition problems

Surjective partitions

Each part is non-empty

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M not necessarily symmetric

Partitioning digraphs G

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Partitioning digraphs G

Vertices of the graph G are equipped with lists

Each vertex placed in a part on its list

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Surjective partitions

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Partitioning digraphs G

Vertices of the graph G are equipped with lists

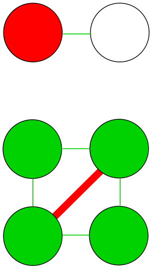
Each vertex placed in a part on its list

Edge-coloured complete graphs G

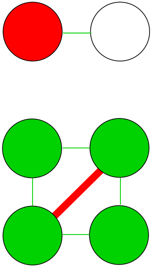
M records which colours are allowed

Study of perfect graphs

Study of perfect graphs



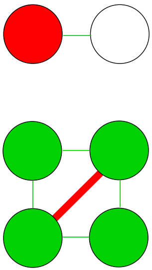
Study of perfect graphs



Split graphs

Polynomial time recognition by Foldes-Hammer 1973

Study of perfect graphs



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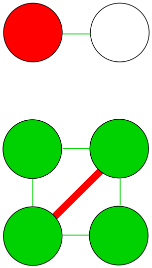

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Quasi-polynomial with lists, Feder-Hell-Klein-Motwani 2003

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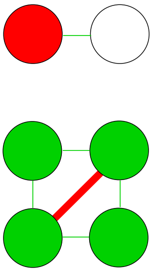

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Polynomial with lists, Figueiredo-Klein-Kohayakawa-Reed 2007

Complexity of M-partition

Given M

Is the M-partition problem NP-complete or polynomial time solvable?

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In the digraph variant, if M has no 1's

The Dichotomy Conjecture of CSP: YES

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Each M-partition problem is NP-complete or solvable in polynomial time

Complexity of M-partition

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Is the M-partition problem NP-complete or polynomial time solvable?

In the digraph variant, if M has no 1's

The Dichotomy Conjecture of CSP: YES
Each M-partition problem is NP-complete or solvable in polynomial time

Feder-Hell 2006

Each M-partition problem is NP-complete or solvable in quasi-polynomial time

Complexity of M-partition for restricted graphs

Complexity of M-partition for restricted graphs

Given M

Is the M-partition problem for perfect graphs NP-complete or polynomial time solvable?

Complexity of *M*-partition for restricted graphs

Given *M*

Is the *M*-partition problem for perfect graphs NP-complete or polynomial time solvable?

Feder-Hell 2006

Each constraint satisfaction problem is polynomial time equivalent to some *M*-partition problem for perfect graphs

Complexity of M-partition for restricted graphs

Given M

Is the M-partition problem for perfect graphs NP-complete or polynomial time solvable?

Feder-Hell 2006

Each constraint satisfaction problem is polynomial time equivalent to some M-partition problem for perfect graphs

($\forall\ H\ \exists\ M$ with CSP(H) $\sim_P$ M-partition)

Complexity of M-partition for restricted graphs

Cographs

Every M-partition problem can be solved in linear time

Complexity of M-partition for restricted graphs

Cographs

Every M-partition problem can be solved in linear time

Courcelle-Makowsky-Rotics 2000
Monadic second order logic description of M-partition: $\exists A \exists B \ldots$

Complexity of M-partition for restricted graphs

Cographs

Every M-partition problem can be solved in linear time

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Chordal graphs

Complexity of M-partition for restricted graphs

Cographs

Every M-partition problem can be solved in linear time

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Chordal graphs

$$\begin{pmatrix} 0 & * & * & * & * \\ * & 0 & * & * & * \\ * & * & 0 & * & * \\ * & * & * & 0 & * \\ * & * & * & * & 0 \end{pmatrix}$$

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Polynomial
Gavril 1971

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$$\begin{pmatrix} 1 & * & * & * & * \\ * & 1 & * & * & * \\ * & * & 1 & * & * \\ * & * & * & 1 & * \\ * & * & * & * & 1 \end{pmatrix}$$

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Chordal graphs

$$\begin{pmatrix} 0 & * & * & * & * \\ * & 0 & * & * & * \\ * & * & 0 & * & * \\ * & * & * & 1 & * \\ * & * & * & * & 1 \end{pmatrix}$$

Complexity of *M*-partition for restricted graphs

Cographs

Every *M*-partition problem can be solved in linear time

Courcelle-Makowsky-Rotics 2000
Monadic second order logic description of *M*-partition: $\exists A \exists B \ldots$

Chordal graphs

$$\begin{pmatrix} 0 & * & * & * & * \\ * & 0 & * & * & * \\ * & * & 0 & * & * \\ * & * & * & 1 & * \\ * & * & * & * & 1 \end{pmatrix}$$

Polynomial
(Gimbel-Kratsch-Stewart 1994)

Complexity of M-partition for chordal graphs

Chordal graphs

Is every M-partition problem polynomial?

Complexity of M-partition for chordal graphs

Chordal graphs

Is every M-partition problem polynomial?

Chordal graphs

If M has 0 diagonal then the M-partition problem is polynomial

Feder-Hell-Klein-Nogueira-Protti 2005

Complexity of M-partition for chordal graphs

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Example

$$\begin{pmatrix} 0 & * & * & 1 & 1 \\ * & 0 & 1 & 1 & * \\ * & 1 & 0 & * & 0 \\ 1 & 1 & * & 0 & * \\ 1 & * & 0 & * & 0 \end{pmatrix}$$

Complexity of M-partition for chordal graphs

Chordal graphs

Is every M-partition problem polynomial?

Chordal graphs

If M has 1 diagonal then the M-partition problem is polynomial

Feder-Hell-Klein-Nogueira-Protti 2005

Example

$$\begin{pmatrix} 1 & * & * & 1 & 1 \\ * & 1 & 1 & 1 & * \\ * & 1 & 1 & * & 0 \\ 1 & 1 & * & 1 & * \\ 1 & * & 0 & * & 1 \end{pmatrix}$$

Complexity of *M*-partition for chordal graphs

Chordal graphs

Is every *M*-partition problem polynomial?

A crossed matrix

$$\begin{pmatrix} 0 & * & * & 1 & 1 \\ * & 0 & 1 & 1 & 0 \\ * & 1 & 0 & * & 0 \\ 1 & 1 & * & 1 & * \\ 1 & 0 & 0 & * & 1 \end{pmatrix}$$

Each 0 and each 1 lies in a red row or column without *

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Chordal graphs

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Complexity of M-partition for chordal graphs

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Not a crossed matrix

$$\begin{pmatrix} 0 & * & * & 1 & 1 \\ * & 0 & 1 & 1 & * \\ * & 1 & 0 & * & 0 \\ 1 & 1 & * & 1 & * \\ 1 & * & 0 & * & 1 \end{pmatrix}$$

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Complexity of M-partition for chordal graphs

Chordal graphs

Is every M-partition problem polynomial?

Crossed matrices

If M is crossed then the M-partition problem is polynomial

Complexity of *M*-partition for chordal graphs

Chordal graphs

Is every *M*-partition problem polynomial?

Crossed matrices

If *M* is crossed then the *M*-partition problem is polynomial

But

There exist non-crossed matrices *M* for which the *M*-partition problem is NP-complete.

Feder-Hell-Klein-Nogueira-Protti 2005

Three Possibilities

The M-partition problem

Three Possibilities

The M-partition problem

- NP-complete

Three Possibilities

The M-partition problem

- NP-complete $\begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$

Three Possibilities

The M-partition problem

- NP-complete $\begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$
- polynomial time solvable but with infinitely many minimal obstructions

Three Possibilities

The M-partition problem

- NP-complete $\begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$
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Three Possibilities

The M-partition problem

- NP-complete $\begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$
- polynomial time solvable but with infinitely many minimal obstructions $\begin{pmatrix} 0 & * \\ * & 0 \end{pmatrix}$
- with a finite set of minimal obstructions

Three Possibilities

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- NP-complete $\begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$
- polynomial time solvable but with infinitely many minimal obstructions $\begin{pmatrix} 0 & * \\ * & 0 \end{pmatrix}$
- with a finite set of minimal obstructions $\begin{pmatrix} 0 & * \\ * & 1 \end{pmatrix}$

Three Possibilities

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- NP-complete $\begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$
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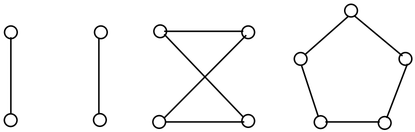
- with a finite set of minimal obstructions $\begin{pmatrix} 0 & * \\ * & 1 \end{pmatrix}$



New Hope

The M-partition problem

For which matrices M does it have a finite set of minimal obstructions?

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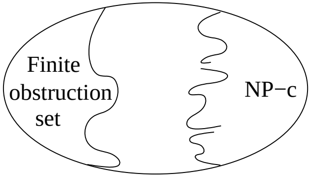

New Hope

The *M*-partition problem

For which matrices M does it have a finite set of minimal obstructions?

Unfriendly matrices *M*

If M contains $\begin{pmatrix} 0 & * \\ * & 0 \end{pmatrix}$ or $\begin{pmatrix} 1 & * \\ * & 1 \end{pmatrix}$, then the M-partition problem has infinitely many minimal obstructions

New Hope

The M-partition problem

For which matrices M does it have a finite set of minimal obstructions?

Unfriendly matrices M

If M contains $\begin{pmatrix} 0 & * \\ * & 0 \end{pmatrix}$ or $\begin{pmatrix} 1 & * \\ * & 1 \end{pmatrix}$, then the M-partition problem has infinitely many minimal obstructions

Feder-Hell-Xie 2007

New Hope

The *M*-partition problem

For which matrices M does it have a finite set of minimal obstructions?

Friendly matrices *M* of size at most 5

If M does not contain $\begin{pmatrix} 0 & * \\ * & 0 \end{pmatrix}$ or $\begin{pmatrix} 1 & * \\ * & 1 \end{pmatrix}$, then the M-partition problem has finitely many minimal obstructions

Feder-Hell-Xie 2007

New Hope

The *M*-partition problem

For which matrices *M* does it have a finite set of minimal obstructions?

BUT

Friendly matrices *M*

There is a friendly matrix *M* of size 6 such that the *M*-partition problem has infinitely many minimal obstructions

New Hope

The *M*-partition problem

For which matrices *M* does it have a finite set of minimal obstructions?

BUT

Friendly matrices *M*

There is a friendly matrix *M* of size 6 such that the *M*-partition problem has infinitely many minimal obstructions

There is even a friendly matrix *M* of size 9 such that the *M*-partition problem is NP-complete

Feder-Hell-Xie 2007

Restricted Graphs

If M has no 1's

A perfect graph G has an M-partition $\iff$ it contains no induced $K_{c(M)+1}$

Restricted Graphs

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$c(M)$ is the maximum clique size of the graph with "adjacency matrix" M

If M has no 1's

A perfect graph G has an M-partition $\iff$ it contains no induced $K_{c(M)+1}$

$c(M)$ is the maximum clique size of the graph with "adjacency matrix" M

Similarly for matrices without 0

Normal matrices

$$\begin{pmatrix} 0 & a_1 & a_2 & c_1 & c_2 \\ a_1 & 0 & a_3 & c_3 & c_4 \\ a_2 & a_3 & 0 & c_5 & c_6 \\ c_1 & c_3 & c_5 & 1 & b_1 \\ c_2 & c_4 & c_6 & b_1 & 1 \end{pmatrix}$$

Normal matrices

$$\begin{pmatrix} 0 & a_1 & a_2 & c_1 & c_2 \\ a_1 & 0 & a_3 & c_3 & c_4 \\ a_2 & a_3 & 0 & c_5 & c_6 \\ c_1 & c_3 & c_5 & 1 & b_1 \\ c_2 & c_4 & c_6 & b_1 & 1 \end{pmatrix}$$

Either all or none of a_1, a_2, a_3 are $*$
Either all or none of $c_1, c_2, \ldots, c_6$ are $*$
Etc for the b_i

Normal matrices

$$\begin{pmatrix} 0 & a_1 & a_2 & c_1 & c_2 \\ a_1 & 0 & a_3 & c_3 & c_4 \\ a_2 & a_3 & 0 & c_5 & c_6 \\ c_1 & c_3 & c_5 & 1 & b_1 \\ c_2 & c_4 & c_6 & b_1 & 1 \end{pmatrix}$$

Either all or none of a_1, a_2, a_3 are $*$
Either all or none of $c_1, c_2, \ldots, c_6$ are $*$
Etc for the b_i
(A normal matrix is crossed.)

Normal matrices

$$\begin{pmatrix} 0 & a_1 & a_2 & c_1 & c_2 \\ a_1 & 0 & a_3 & c_3 & c_4 \\ a_2 & a_3 & 0 & c_5 & c_6 \\ c_1 & c_3 & c_5 & 1 & b_1 \\ c_2 & c_4 & c_6 & b_1 & 1 \end{pmatrix}$$

Feder-Hell 2006

If M is normal, then the M-partition problem has finitely many perfect minimal obstructions

M-partition for Restricted Graphs

Cographs

Every *M*-partition problem has finitely many minimal obstruct's

M-partition for Restricted Graphs

Cographs

Every M-partition problem has finitely many minimal obstruct's

Damaschke 2004
Feder-Hell-Hochstättler 2006

Chordal graphs

Chordal graphs

Example

$$\begin{pmatrix} 0 & * & * & * & * \\ * & 0 & * & * & * \\ * & * & 0 & * & * \\ * & * & * & 1 & * \\ * & * & * & * & 1 \end{pmatrix}$$

Chordal graphs

Example

$$\begin{pmatrix} 0 & * & * & * & * \\ * & 0 & * & * & * \\ * & * & 0 & * & * \\ * & * & * & 1 & * \\ * & * & * & * & 1 \end{pmatrix}$$

Hell-Klein-Nogueira-Protti 2004

A chordal graph G can be partitioned into ℓ independent sets and m cliques if and only if it does not contain an induced $(m+1)K_{\ell+1}$

Chordal graphs

Chordal graphs

Which matrices M yield M-partition problems with finitely many chordal minimal obstructions?

Chordal graphs

Small matrices with zero diagonal

$$M = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$$

Small matrices with zero diagonal

$$M = \begin{pmatrix} 0 & a & * \\ a & 0 & c \\ * & c & 0 \end{pmatrix}$$

Chordal graphs

Small matrices with zero diagonal

$$M = \begin{pmatrix} 0 & * & * \\ * & 0 & c \\ * & c & 0 \end{pmatrix}$$

Chordal graphs

Small matrices with zero diagonal

$$M = \begin{pmatrix} 0 & * & * \\ * & 0 & 0 \\ * & 0 & 0 \end{pmatrix}$$

Small matrices with zero diagonal

$$M = \begin{pmatrix} 0 & * & * \\ * & 0 & 1 \\ * & 1 & 0 \end{pmatrix}$$

Chordal graphs

Small matrices with zero diagonal

$$M = \begin{pmatrix} 0 & 1 & * \\ 1 & 0 & 1 \\ * & 1 & 0 \end{pmatrix}$$

Chordal graphs

Interesting small matrices

$$\begin{pmatrix} 0 & * & * \\ * & 0 & 1 \\ * & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & * \\ 1 & 0 & 1 \\ * & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & * \\ 0 & 1 & 0 \\ * & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & * \\ 0 & 1 & * \\ * & * & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & * & * \\ * & 0 & 1 \\ * & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & * & * \\ * & 0 & 0 \\ * & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & * & 0 \\ * & 1 & * \\ 0 & * & 0 \end{pmatrix} \begin{pmatrix} 1 & * & 1 \\ * & 1 & * \\ 1 & * & 0 \end{pmatrix}$$

Eight interesting theorems

Chordal graphs

Small matrices

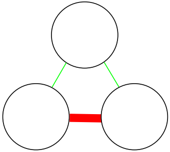
For matrices of size smaller than 4, only

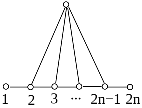
$$M = \begin{pmatrix} 0 & * & * \\ * & 0 & 1 \\ * & 1 & 0 \end{pmatrix} \text{ and } M = \begin{pmatrix} 0 & * & * \\ * & 0 & 1 \\ * & 1 & 1 \end{pmatrix}$$

have an M-partition problem with infinitely many chordal minimal obstructions

Feder - Hell - Nekooei-Rizi 2011

Chordal graphs





Small matrices

For

$$M = \begin{pmatrix} 0 & 1 & * \\ 1 & 0 & 1 \\ * & 1 & 0 \end{pmatrix}$$

the M-partition problem has exactly three chordal minimal obstructions

Feder - Hell - Nekooei-Rizi 2011

Chordal graphs

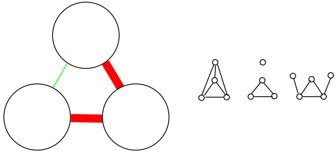
Small matrices

For

$$M = \begin{pmatrix} 0 & 1 & * \\ 1 & 0 & 1 \\ * & 1 & 0 \end{pmatrix}$$

the *M*-partition problem has exactly three chordal minimal obstructions

Feder - Hell - Nekooei-Rizi 2011



Chordal graphs

Small matrices

For

$$M = \begin{pmatrix} 1 & 0 & * \\ 0 & 1 & * \\ * & * & 1 \end{pmatrix}$$

the M-partition problem has finitely many chordal minimal obstructions

Feder - Hell - Nekooei-Rizi 2011

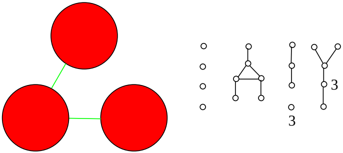
Small matrices

For

$$M = \begin{pmatrix} 1 & 0 & * \\ 0 & 1 & * \\ * & * & 1 \end{pmatrix}$$

the M-partition problem has finitely many chordal minimal obstructions

Feder - Hell - Nekooei-Rizi 2011



Chordal graphs

What's next?

What's next?

- Which matrices M lead to finitely many chordal minimal obstructions?

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- Are they more numerous than those that lead to infinitely many?

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Finitely many

Chordal graphs

What's next?

- Which matrices M lead to finitely many chordal minimal obstructions?
- Are they more numerous than those that lead to infinitely many?

Finitely many

Matrices with only two kinds of entries (Feder-Hell 2008)

Chordal graphs

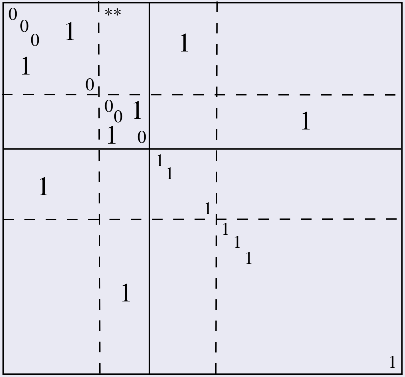
Small matrices

$$M = \begin{pmatrix} 0 & * & * \\ * & 0 & 1 \\ * & 1 & 0 \end{pmatrix} \text{ and } M = \begin{pmatrix} 0 & * & * \\ * & 0 & 1 \\ * & 1 & 1 \end{pmatrix}$$

have an M-partition problem with infinitely many chordal minimal obstructions

Chordal graphs

Infinitely many



Chordal graphs

k diagonal 0's and k diagonal 1's

Of the total T, more than $\sqrt{T}$ have infinitely many chordal minimal obstructions.

Interesting Open Case

Diagonal of 1's

Always finitely many chordal minimal obstructions?

(Or at most one 0)