Induced paths of given parity in planar graphs

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GROW, KAIST, Daejeon, South Korea, October 2011

Why induced?

How to find an odd (not necessarily induced) path between two vertices?

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Bipartite?

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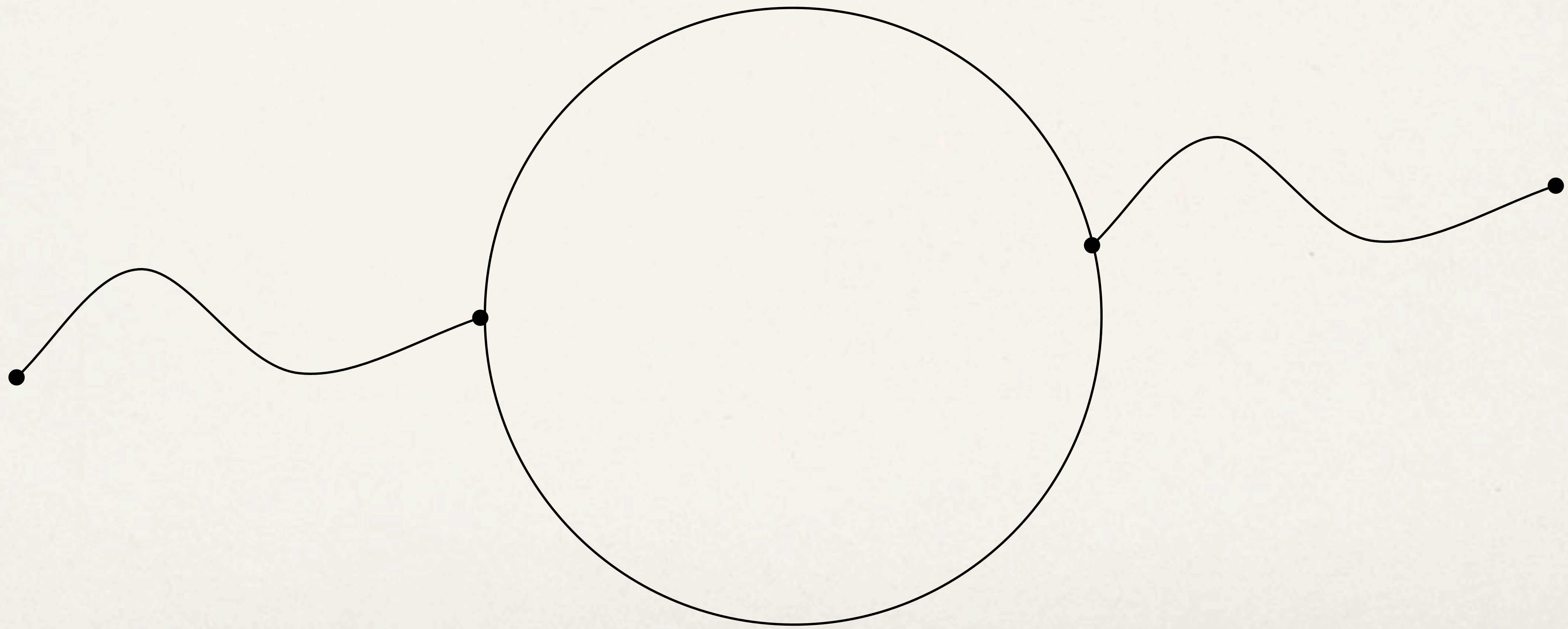

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LaPaugh and Papadimitriou, 1984

For fixed integers p, q, there is a linear-time algorithm testing whether there is a path of length p mod q between a given pair of vertices.

Arkin, Papadimitriou, Yanakakis, 1991

Why planar?

Testing whether there is an induced odd/even path between a given pair of vertices is NP-complete.

Bienstock, 1991

Before…

- chordal graphs

Arikati and Peled, 1993

- circular arc

Arikati, Rangan, Manacher, 1991

- comparability and cocomparability

Satyan and Rangan, 1996

- perfectly orientable

Arikati and Peled, 1996

- planar perfect

Sampio and Sales, 2001

- claw-free

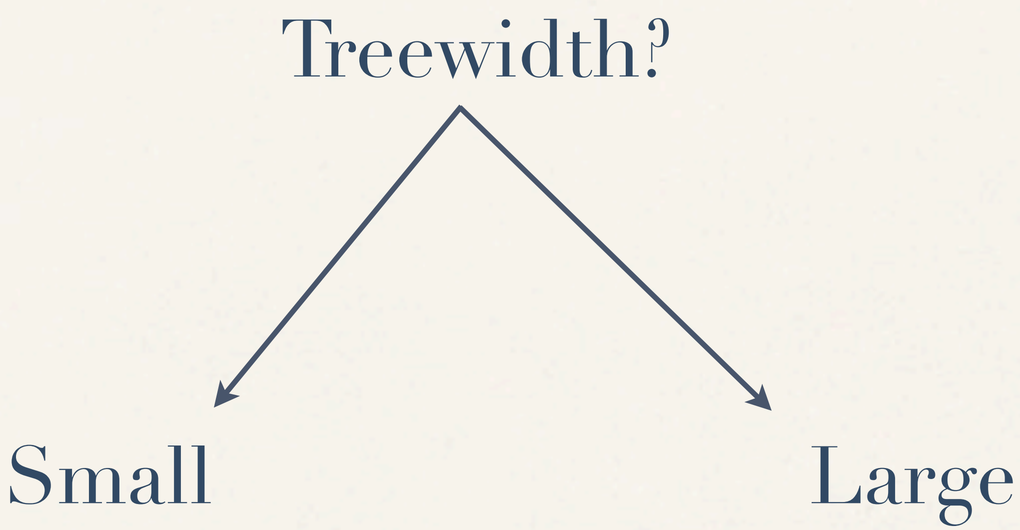
van ‘t Hof, K., Paulusma, 2009

Solution for planar graphs

Outline of the solution

Treewidth?

Outline of the solution



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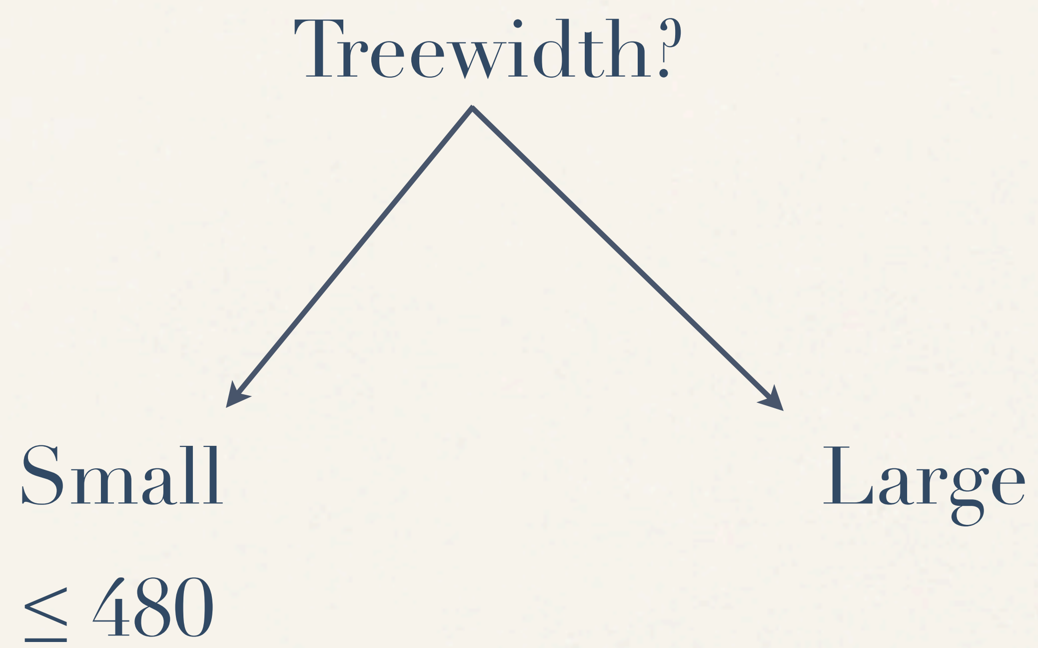

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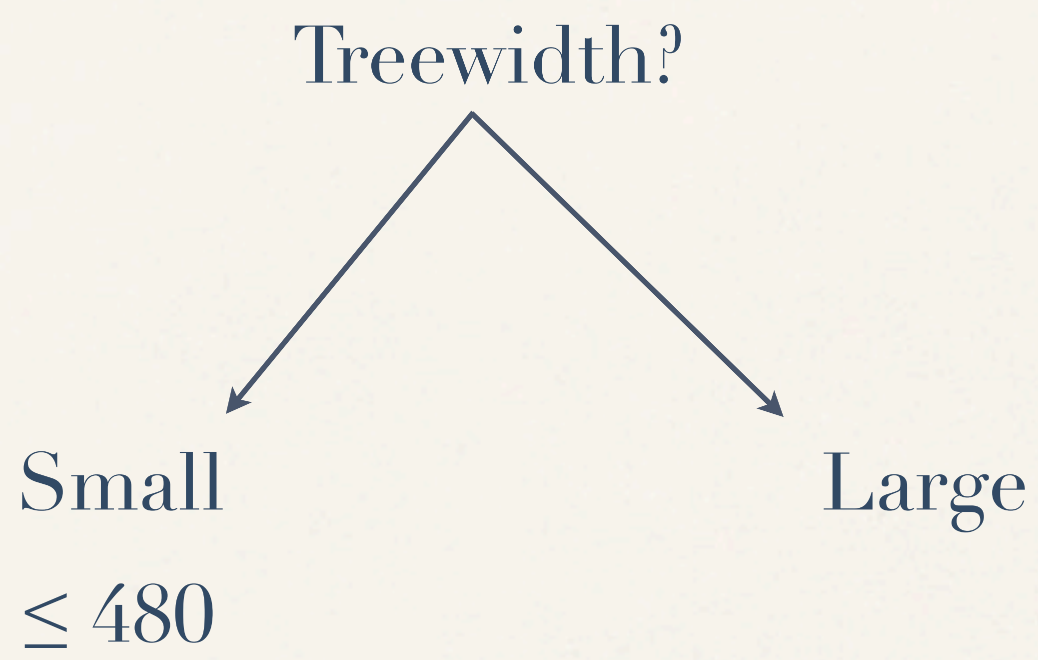

- small treewidth: solve by dynamic programming

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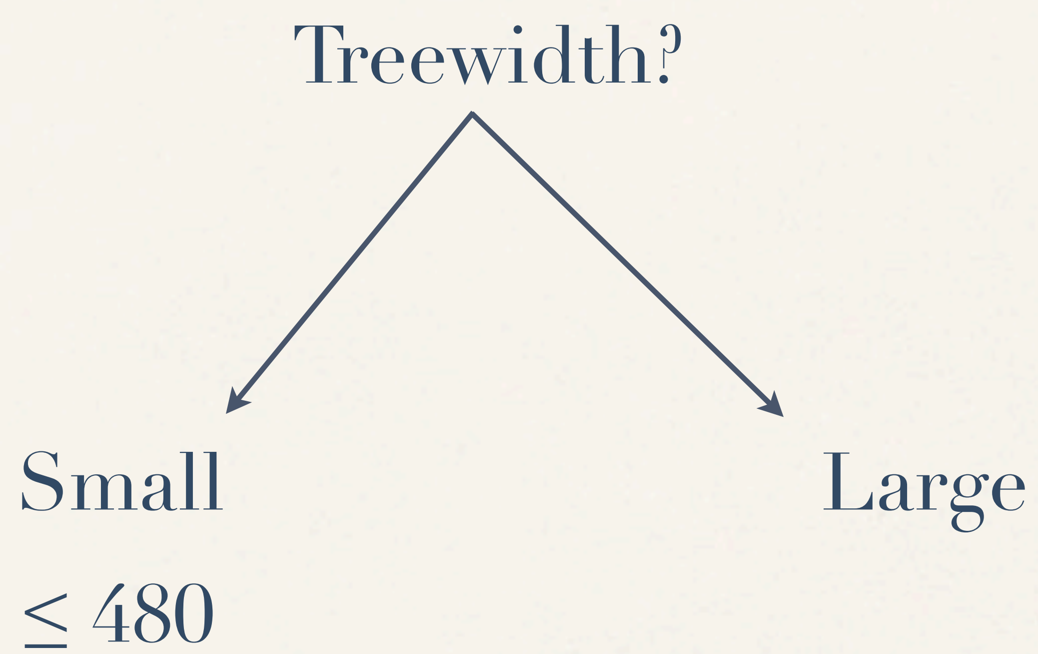

- small treewidth: solve by dynamic programming
- large treewidth: find an equivalent smaller instance

Large treewidth

Every planar graph with no $m \times m$ grid minor has tree-width $\leq 6m$-5.

Robertson and Seymour, 1994

Large treewidth

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Every planar graph of "large" treewidth, contains a subdivision of a "large" wall (in a planar way).

Making vertices relevant

There exists a polynomial-time algorithm for induced linkages in planar graphs.

Kawarabayashi and Kobayashi, 2008

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If there is no induced s-v-t-path, v is irrelevant.

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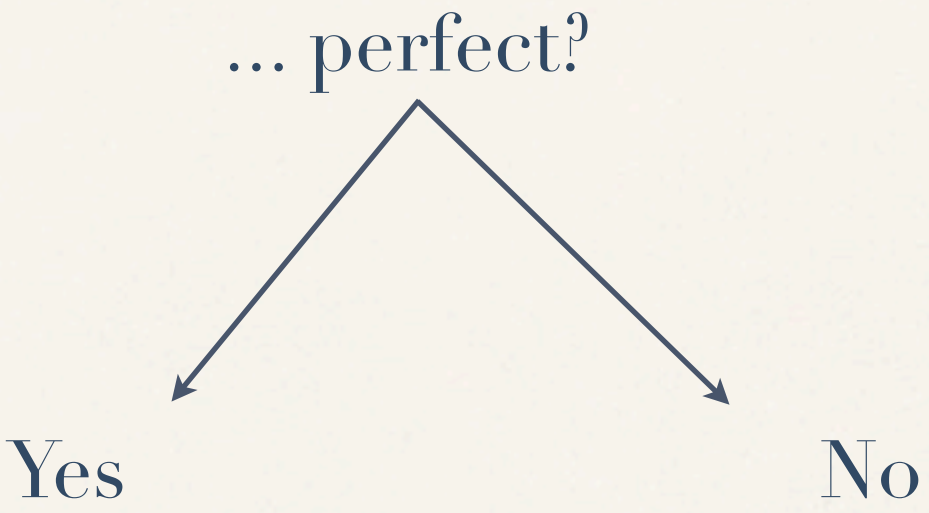
Remove and recurse.

Is the wall …



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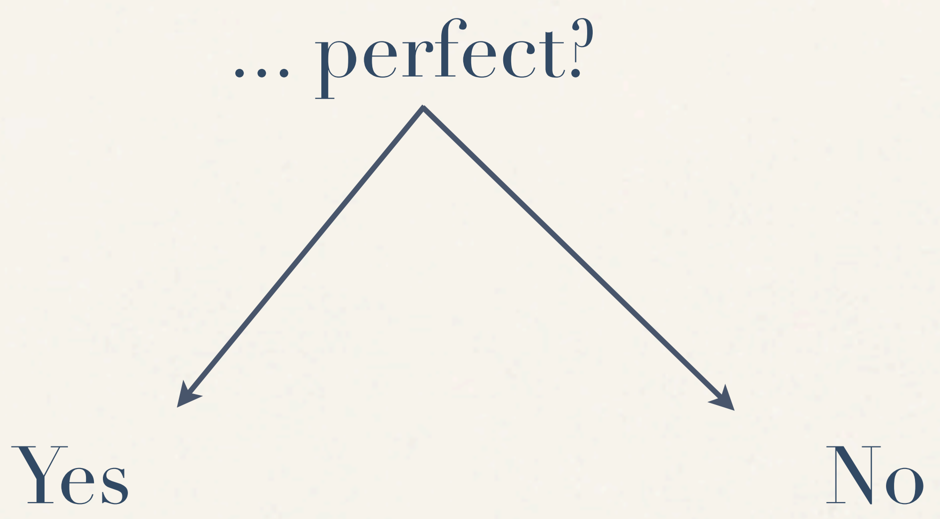

- perfect wall: find an irrelevant vertex

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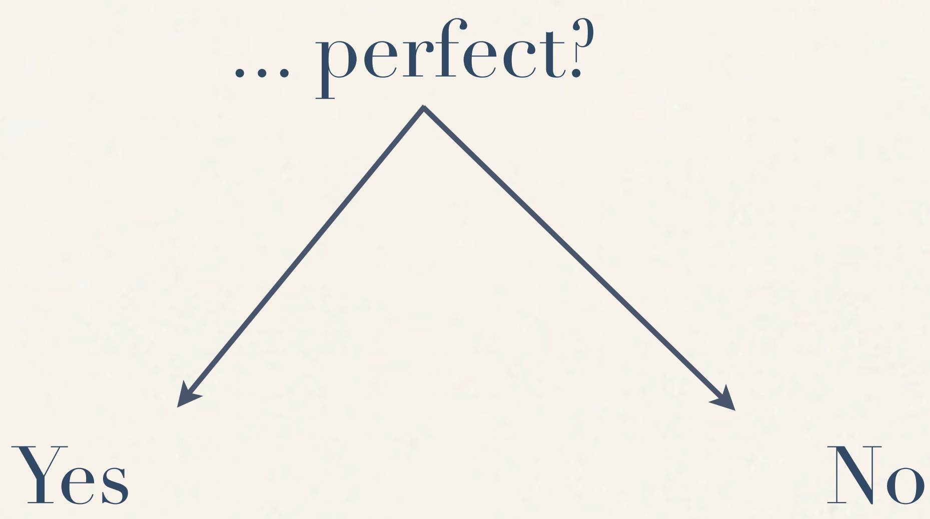
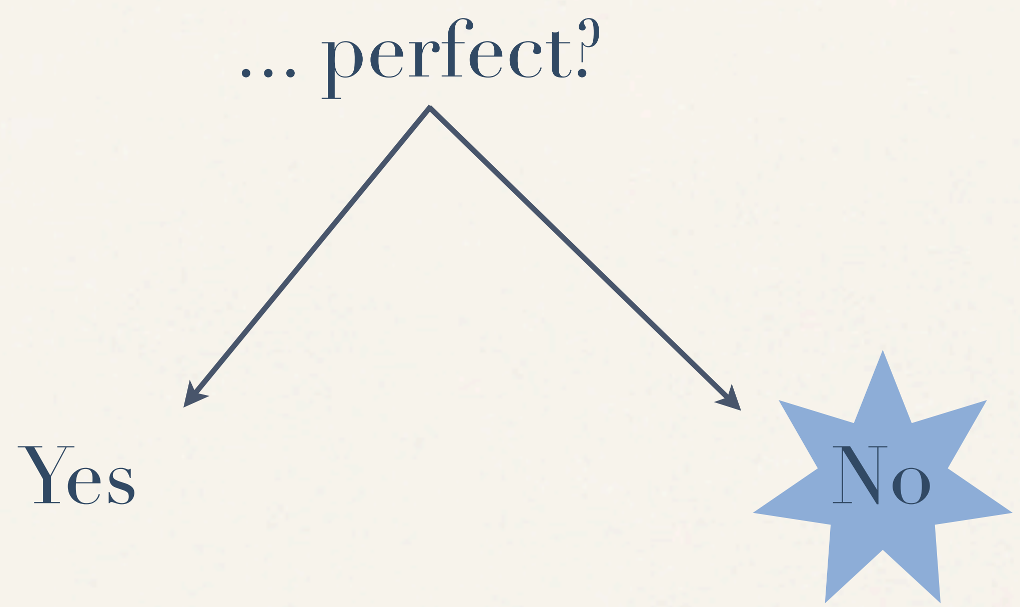


- perfect wall: find an irrelevant vertex
- non-perfect wall: try to connect s and t to an odd hole

Is the wall …



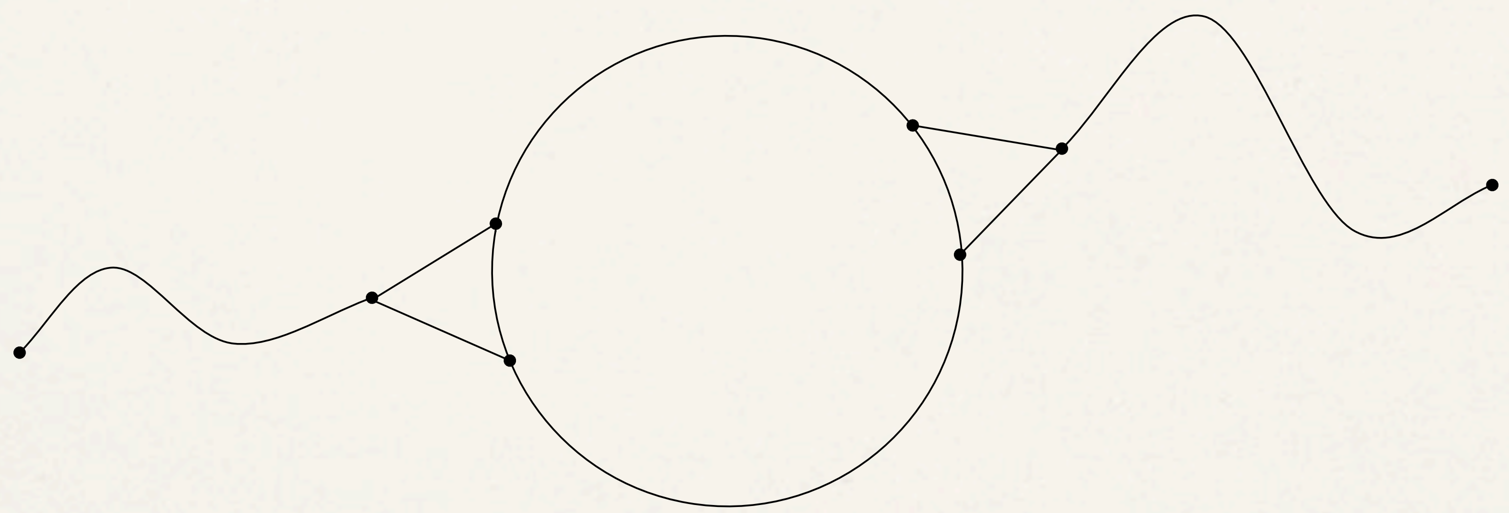
Non-perfect wall

Non-perfect $\implies$ contains an odd hole

Non-perfect wall

Non-perfect $\Longrightarrow$ contains an odd hole

Can we use the approach by LaPaugh and Papadimitriou?



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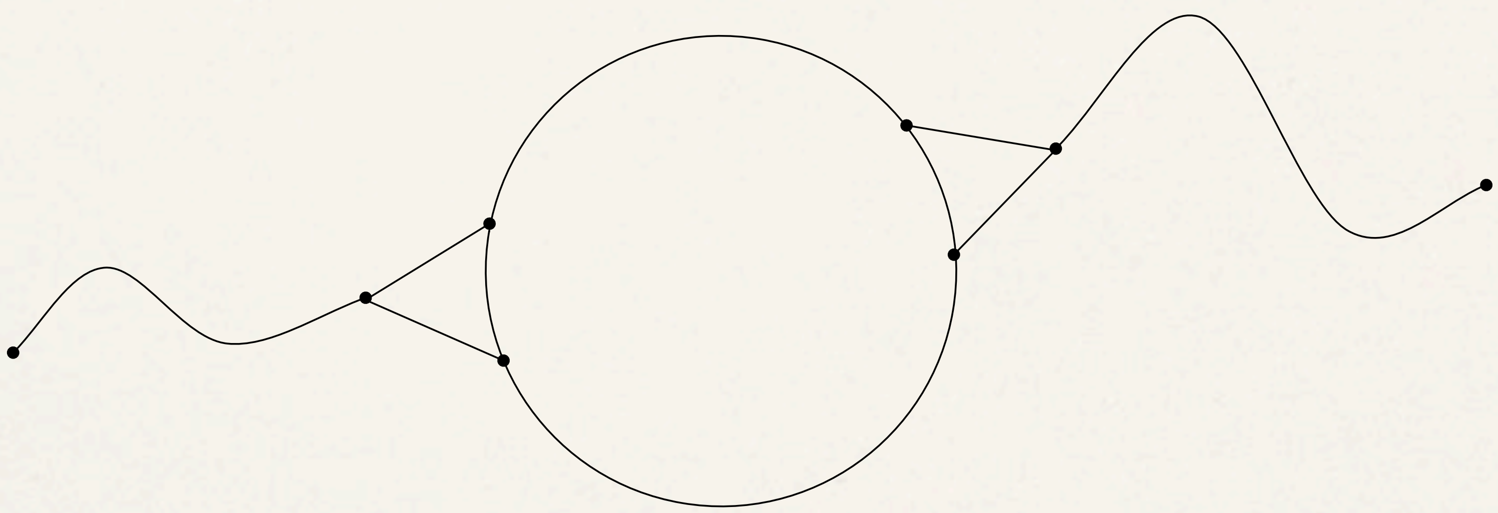

No triangles but ...

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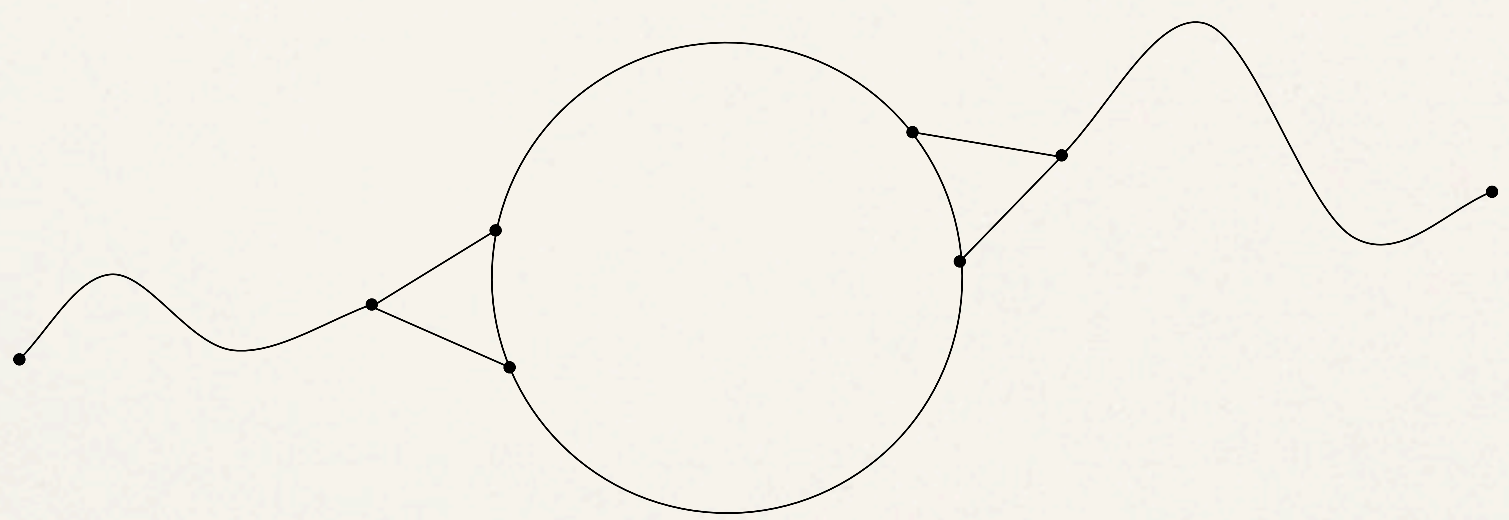

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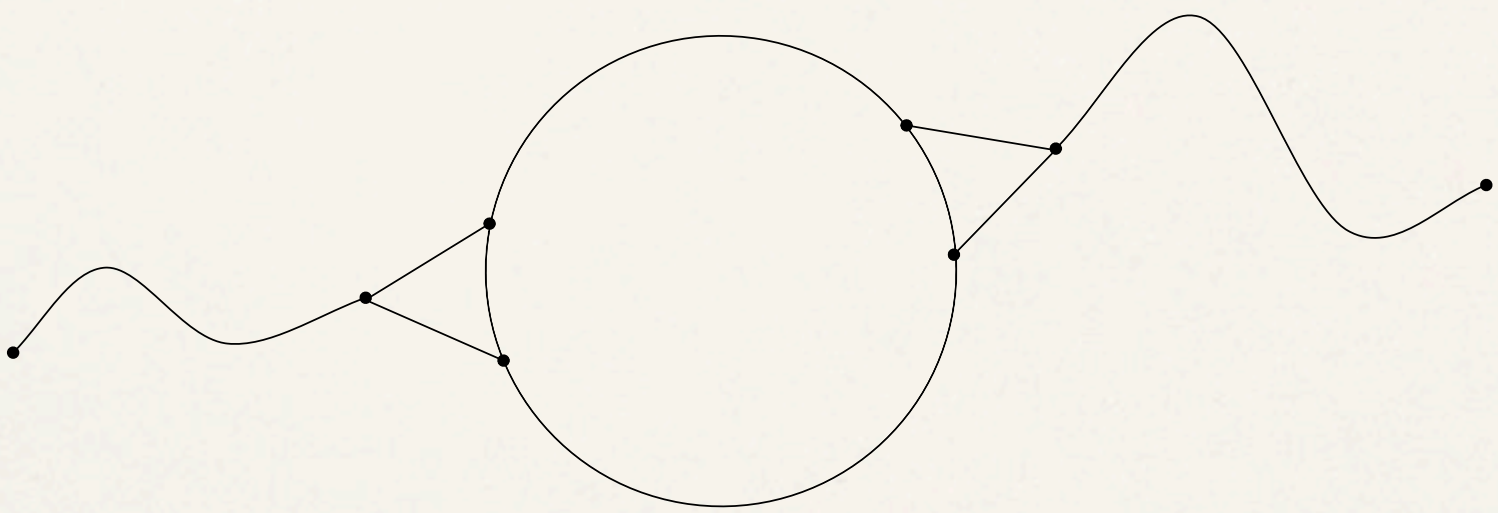
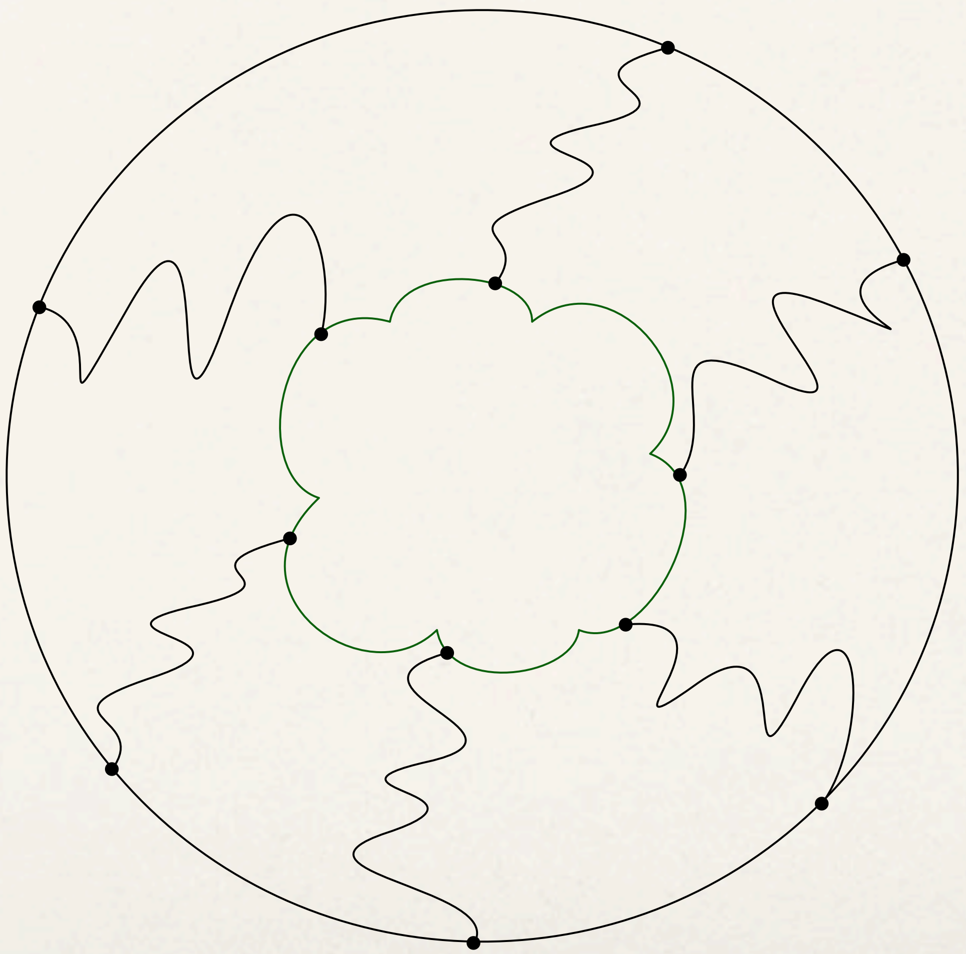


No triangles but the problem is gates + keeping the path induced.

High connectivity

Suppose there exist 6 vertex-disjoint (not necessarily induced) paths between the boundary B of the wall and the odd hole C.



High connectivity

Suppose there exist 6 vertex-disjoint (not necessarily induced) paths between the boundary B of the wall and the odd hole C.

What if there are only 4?

High connectivity

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- either one can “well-connect” B to C, or

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The boundary of the wall can be assumed to be an even cycle.

High connectivity

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- there are 4 vertex-disjoint paths between the boundary B of the wall and the odd hole C, and
- there are 2 mutually induced paths from s and t to two non-adjacent vertices of the boundary B, then

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G is a YES-instance.

Low connectivity

If there is a 1-, 2-, 3-cut between the boundary B of the wall and the odd hole C, then …

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If there is a 1-, 2-, 3-cut between the boundary B of the wall and the odd hole C, then we need to go through some more technicalities.

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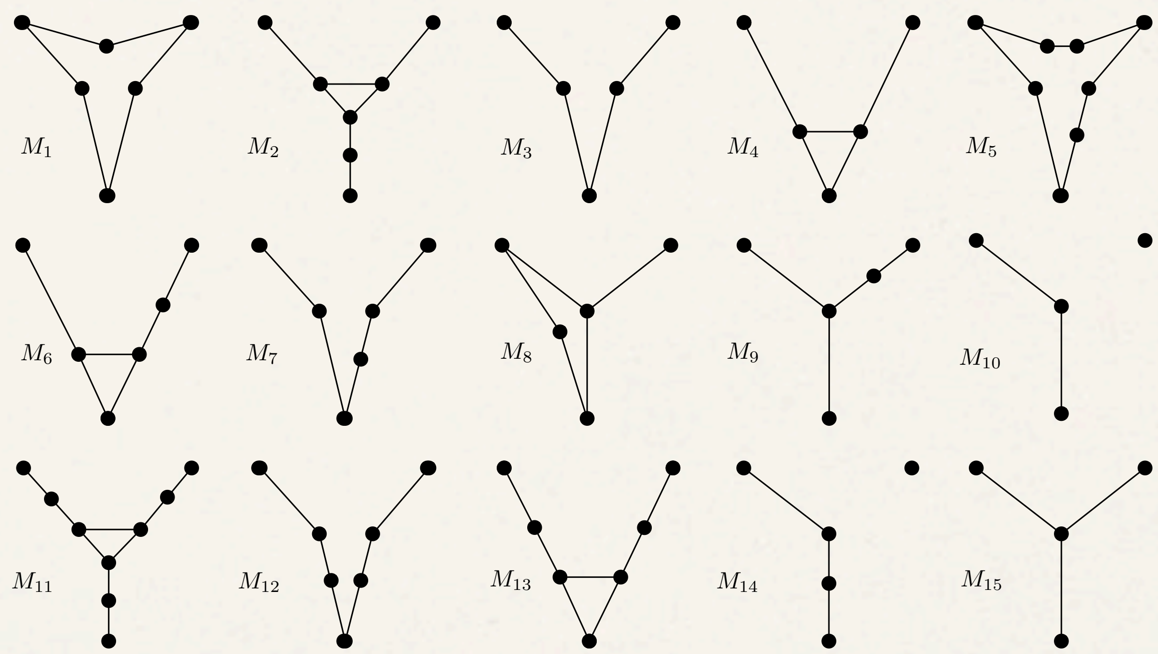
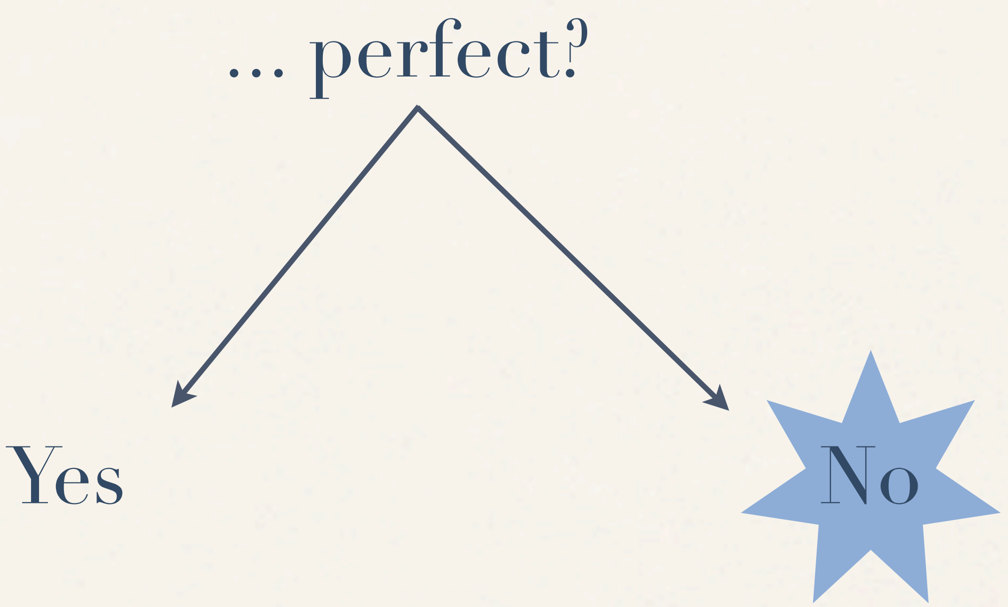


Mimicking gadgets

Is the wall …



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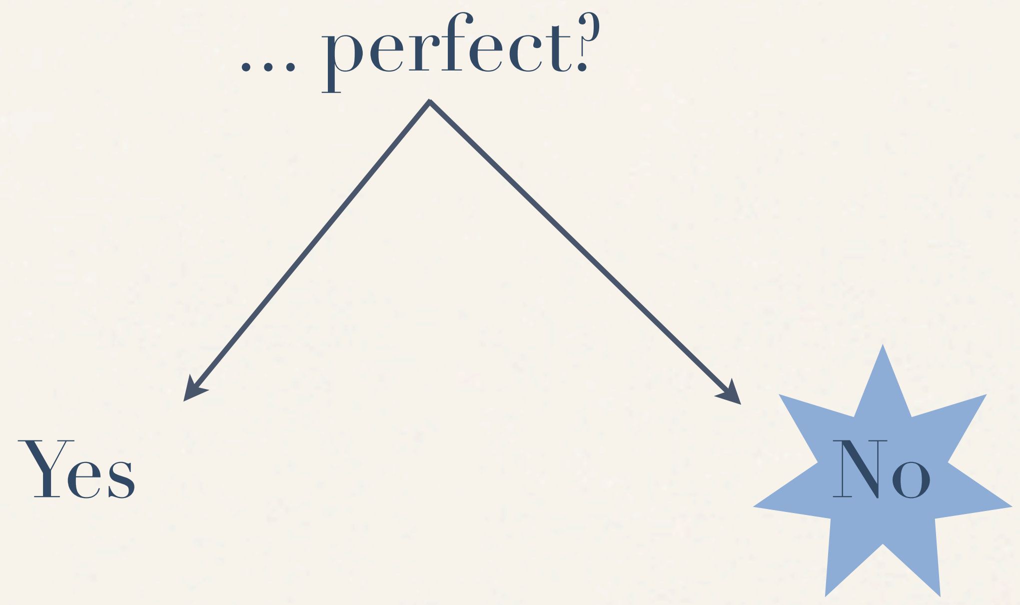

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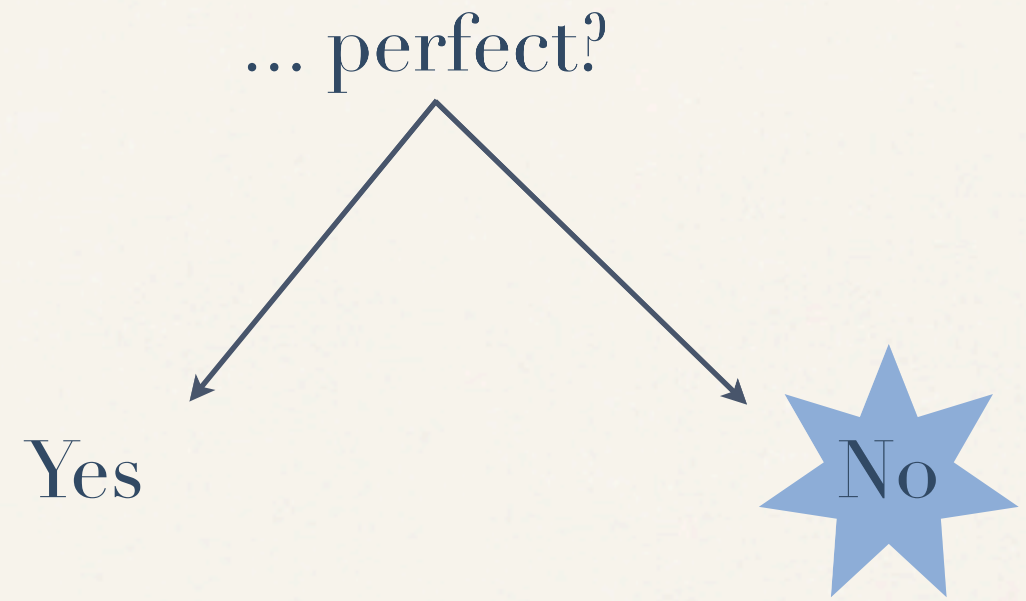
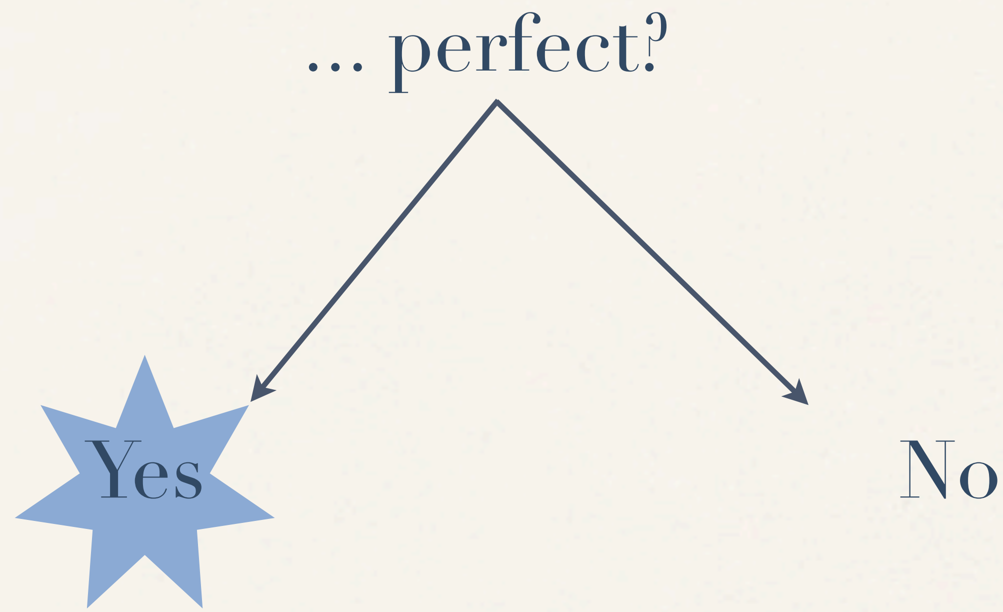


Try to well-connect s and t to the odd hole. If possible, then a YES-instance. If not, find a smaller instance.

Is the wall …



Perfect planar graphs

A planar graph is perfect if and only if it does not contain an odd hole.

Tucker, 1973

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A decomposition theorem for perfect planar graphs.

Hsu, 1987

Hsu decomposition tree

For a planar graph H, we define the rooted Hsu decomposition tree T.

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Hsu decomposition tree

For a planar graph H, we define the rooted Hsu decomposition tree T.

1. Vertices of T are graphs.
2. H is the root of T.
3. If a vertex of T admits one of the four basic cutsets, then its children are the graphs built according to given rules.
4. Leaves of T do not admit any the four basic cutsets.

Four basic cutsets

I. Q is a clique.

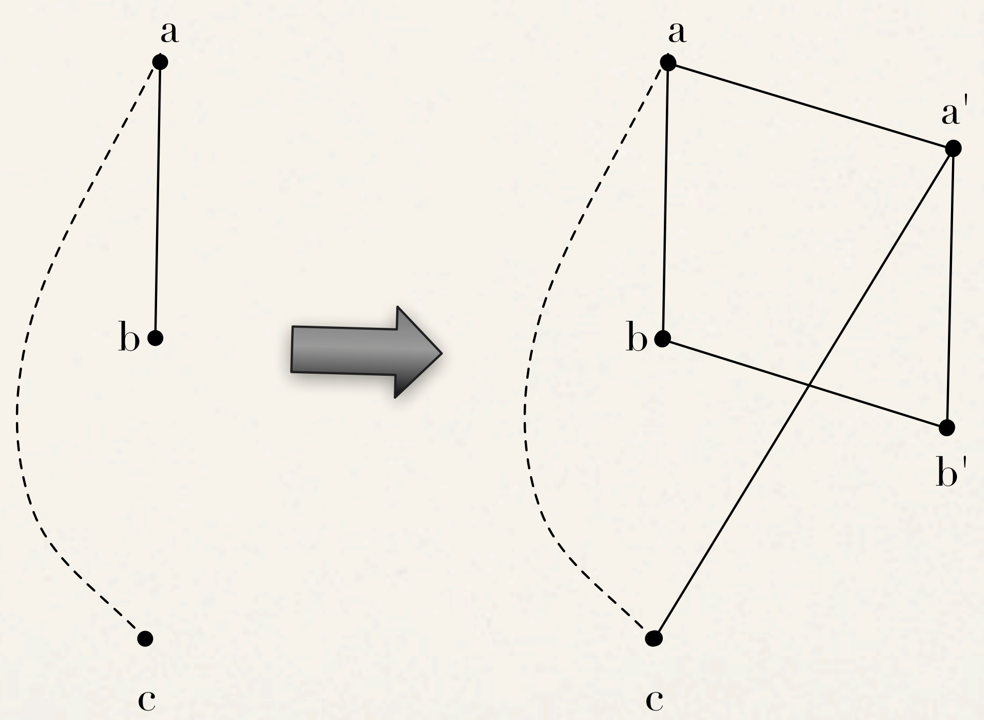
II. Q=$\{a, b\}$ and ad is a non-edge.

III. Q=$\{a, b, c\}$, where ab is an edge, ac is a non-edge, and there is an even induced path a-c-path in each $H[V(C_i) \cup \{a,c\}]$, and an odd induced b-c-path in each $H[V(C_i) \cup \{b,c\}]$.

IV. Q=$\{a, b, c, d\}$ is an induced cycle on 4 vertices with edges ab, bc, cd, da, and there is an even induced path a-c-path in each $H[V(C_i) \cup \{a,c\}]$, and an odd induced b-c-path in each $H[V(C_i) \cup \{b,c\}]$.

Rule III

III. Q={a, b, c}, where ab is an edge, ac is a non-edge, and there is an even induced path a-c-path in each H[V(C_i) ∪ {a,c}], and an odd induced b-c-path in each H[V(C_i) ∪ {b,c}].



b' only if bc is a non-edge

Decomposition of planar perfect graphs

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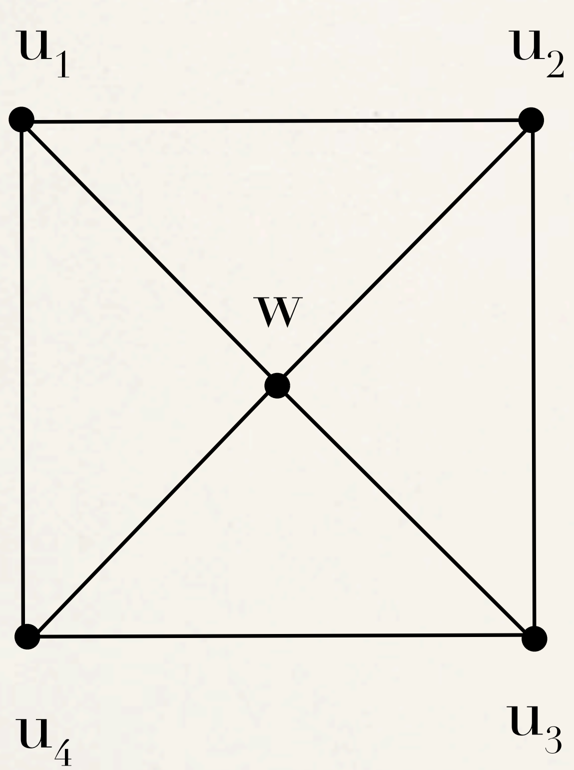
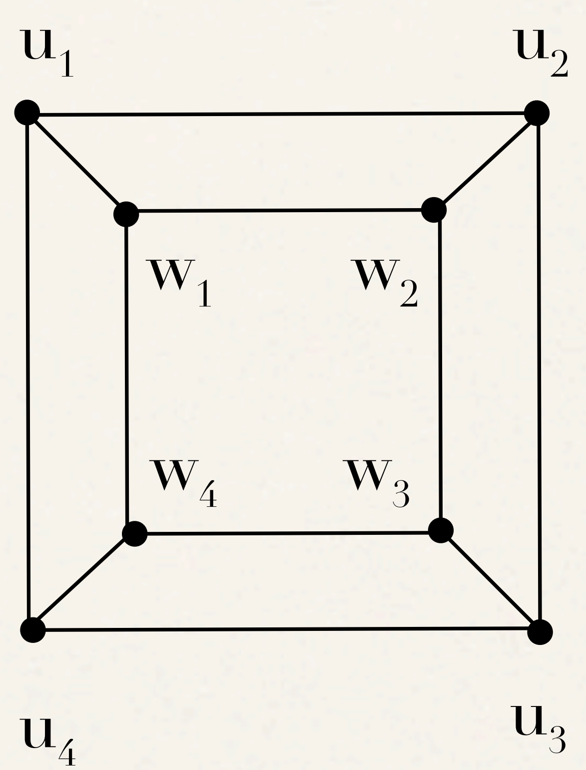
The decomposition tree an be found in cubic time.

Hsu, 1987

Handling C_4-vertices





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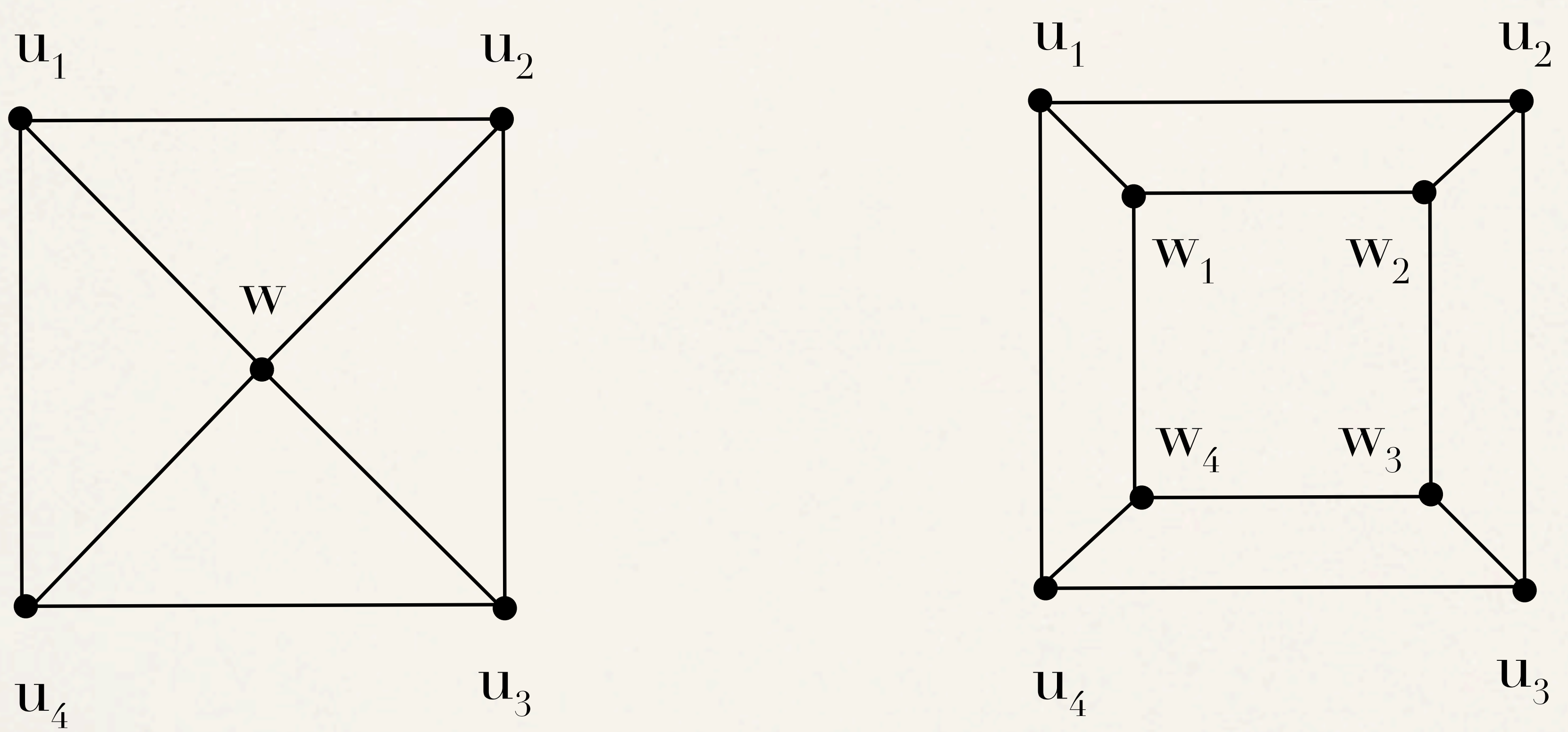


We can assume that the input graph has no C_4-vertices.

Three basic graph classes ...

1. a comparability graph (that contains an independent set of C_4-vertices whose removal leaves the graph bipartite); or

2. a planar line graph of a bipartite graph; or

3. one of ten exceptional graphs (on at most 12 vertices).

… become two

1. a planar bipartite graph; or

2. a planar line graph of a bipartite graph.

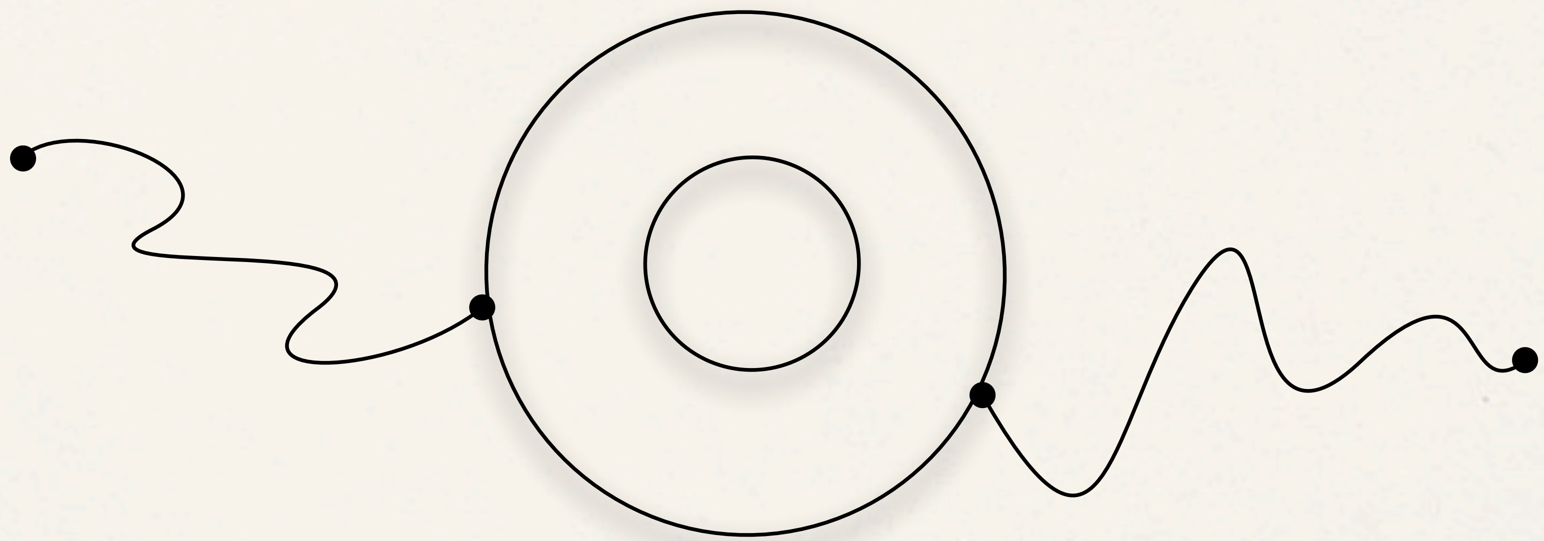
Handling (line graphs of) bipartite graphs

Idea: rerouting the path

Problems: maintaining parity + keeping the path induced

Bipartite graphs

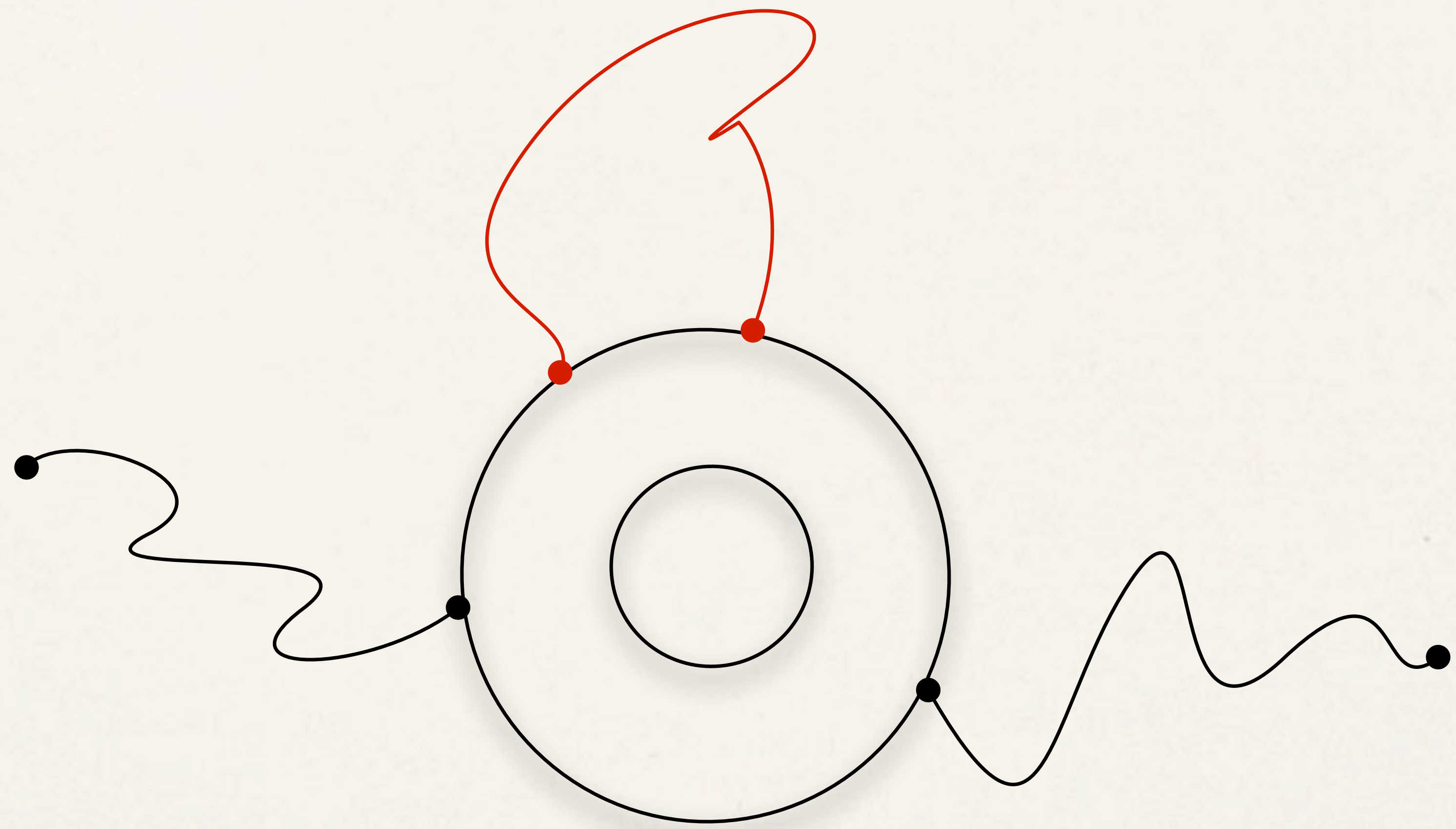
All paths between two vertices of a bipartite graph have the same parity.



Use the inner layer to route the path.

Bipartite graphs

Parity changer

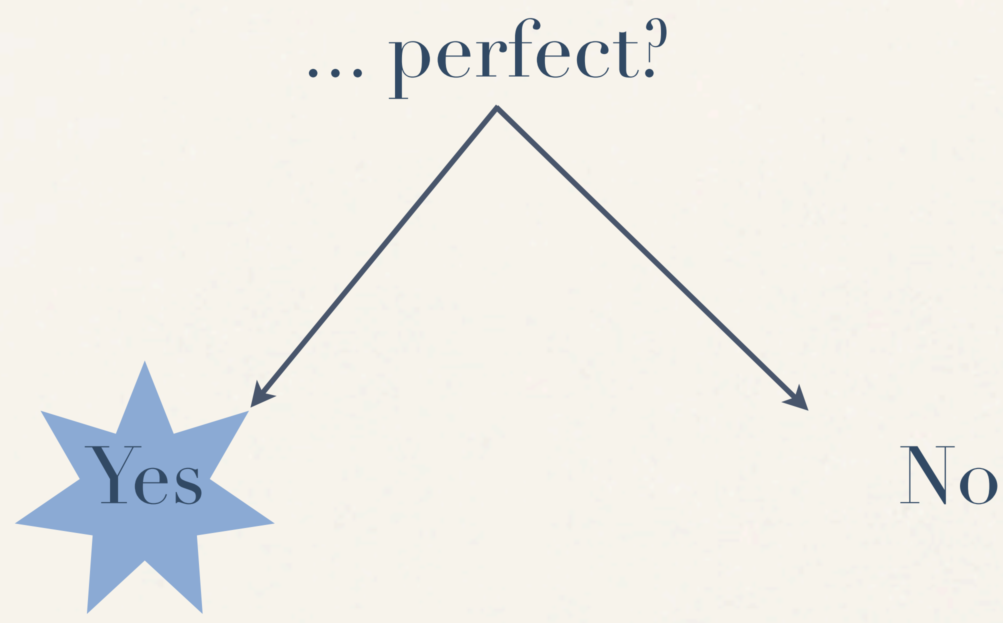


Use the inner layer to route the path and connect to the parity changer.

Line graphs of bipartite graphs

Similar ideas but a bit more difficult to handle.

Is the wall …



Find a smaller instance.

Recap

Small treewidth or large treewidth.

Perfect wall or non-perfect wall.

Find a smaller instance or the input is a YES-instance.

We say NO only in the small treewidth case.

Two paradigms

"topological" vs. "induced"

Thank you!

The Perfect Wall

BSI

Building Science Insights

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BSI-001: The Perfect Wall

By Joseph Lstiburek

Created: 2008/05/20

The perfect wall is an environmental separator—it has to keep the outside out and the inside in. In order to do this the wall assembly has to control rain, air, vapor and heat. In the old days we had one material to do this: rocks. We would pile a bunch or rocks up and have the rocks do it all. But over time rocks lost their appeal. They were heavy and fell down a lot. Heavy means expensive and falling down is annoying. So construction evolved. Today walls need four principal control layers—especially if we don't build out of rocks. They are presented in order of importance:

- a rain control layer
- an air control layer
- a vapor control layer
- a thermal control layer

A point to this importance thing here, if you can't keep the rain out don't waste your time on the air. If you can't keep the air out don't waste your time on the vapor.

Why do we do this?

Parity linkages can be solved in cubic time.

Kawarabayashi, Reed, Wollan, 2011

Perfect connection

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If a graph is perfect and contains and even pair, then the graph obtained by identifying the two vertices is also perfect.

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Fonlupt and Uhry, 1982

Minimal non-perfect graphs contain no even pair.

Meyniel, 1987

Perfect connection

Let’s look for even pairs!

Perfect connection

Let's look for even pairs!

Testing whether two vertices are an even pair, or whether a graph contains an even pair is co-NP-complete.

Bienstock, 1991