SYMMETRY OF A SYMPLECTIC TORIC MANIFOLD

MIKIYA MASUDA

A fundamental result in toric geometry says that there is a bijective correspondence between toric varieties and fans, so all algebrogeometrical information on a toric variety is encoded in the associated fan. Among toric varieties, compact smooth toric varieties, which we call *toric manifolds*, are well studied. If X is a toric manifold, then the group $\mathrm{Aut}(X)$ of automorphisms of X is known to be a (finite dimensional) algebraic group and Demazure introduced a root system for the fan associated with X and proved that it agrees with the root system of the identity component $\mathrm{Aut}^0(X)$ of $\mathrm{Aut}(X)$. He also described the mapping class group $\mathrm{Aut}(X)/\mathrm{Aut}^0(X)$ in terms of the fan associated with X.

A *symplectic toric manifold*, which is a compact symplectic manifold (M, ω) with a Hamiltonian action of a compact torus T where $2 \dim T = \dim M$, is a symplectic counterpart to a toric manifold, but the group $\mathrm{Symp}(M, \omega)$ of symplectomorphisms of (M, ω) is infinite dimensional unlike in the toric case. According to Delzant, symplectic toric manifolds are classified by their moment polytopes. In this talk I introduce a root system $R(P)$ for the moment polytope P. It turns out that any irreducible subsystem of $R(P)$ is of type A and that if G is a compact Lie subgroup of $\mathrm{Symp}(M, \omega)$ containing the torus T, then the root system of G is a subsystem of $R(P)$, so that G is of type A. We can also estimate the finite group G/G^0 in terms of an automorphism group of P.

References

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Department of Mathematics, Graduate School of Science, Osaka City University, Sugimoto, Sumiyoshi-ku, Osaka 558-8585, Japan

E-mail address: masuda@sci.osaka-cu.ac.jp