Redundant decompositions and Time-Frequency Analysis

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This talk is an expository presentation for time-frequncy analysis nad frame. A frame is a sequence of vectors $\{f_i\}_{i\in I}$ in a Hilbert space H that provides robust and stable, but usually non-unique representations of vectors in H. We say that a sequence $\{f_i\}_{i\in I}$ for H if there exist positive constants A and B such that for each $f \in H$,

$$A||f||^2 \le \sum_{i\in I} |\langle f, f_i\rangle|^2 \le B||f||^2.$$

Frames have wide applications in mathematics and engineering, e.g., sampling theory, operator theory, harmonic analysis, nonlinear sparse approximation, pseudo-differential operators, wireless communications, data transmission with erasures, filter banks, signal processing, image processing, geophysics, and quantum computing.

A frame for which the representations of vectors are always unique is a Riesz basis (the image of an orthonormal basis under a continuous, invertible mapping), and conversely. A frame that is not a Riesz basis is redundant in that at least some frame elements may be lost (”erased”), yet still leave a frame. For background on frames we refer to the books [2], [4], [1], or the research-tutorial [3].

Nonunqueness/redundancy is an essential advantage of frames in applications, both in finite and infinite-dimensional spaces. For example, redundant frames have decreased sensitivity to the effect of noise, and since the representations of vectors are nonunique, the space of coefficient choices can be searched to yield sparse representation of data.

Yet, although we know that a redundant frame has extra elements, even the most basic questions about the precise nature of this redundancy are extremely deep and difficult.

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