TIME EVOLUTIONS OF THE STATIONARY SOLUTIONS OF KORTEWEG -DE VRIES EQUATION WITH A POSITIVE FORCING

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Abstract

We study the Korteweg-de Vries (KdV) equation with a positive forcing for a flow of an inviscid and incompressible fluid. The fluid is of a constant density. Steady solutions of FKDV equation are found and numerical stabilities are studied. Moreover, we present the traveling wave solutions and time evolutions of these.

1 Introduction

We consider long nonlinear waves in a two-dimensional flow of inviscid and incompressible fluid of a constant density. The flow is bounded above by a free surface and below by a horizontal rigid boundary containing a bump. The flow is forced by the bump on the bottom with a compact support, which is moving at a constant speed C^*.

Researches in the forced near-critical waves began with meteorological and oceanographic reasons. It is a common feature that atmospheric and oceanic currents are at near critical speeds (Baines [4], Patoine & Warn [1]). Choi, Sun & Shen [11] investigated a flow with a free boundary and developed a refined asymptotic method in order to derive an extended KdV equation with a forcing. Gong & Shen [8] announced many analytic properties of positive solitary wave solutions and numerical results of them for various shapes of forcing corresponding to a flow of a one-layer fluid. Choi [12] concentrated on a negative forcing in case of a flow with a one-layer fluid. He found four positive symmetric or non-symmetric solitary wave-like solutions. Our subject is motivated by these results. In this paper, Numerical stability of two positive solitary wave-like solutions are presented. Based on an asymptotic approach, the first-elevation of the free surface yields forced Korteweg-de Vries equations (fKdV).

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