BOX CONSTRAINED OPTIMIZATION FOR SIGNAL DETECTION IN MIMO CHANNELS

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ABSTRACT

We develop a computationally efficient approximation of the maximum likelihood (ML) detector for 16 quadrature amplitude modulation (16-QAM) in multiple-input multiple-output (MIMO) systems. The detector is based on solving a convex relaxation of the ML problem by a box constrained optimization scheme. Simulation results in a random MIMO system show that this proposed approach outperforms the conventional decorrelator detector and is similar to the semidefinite relaxation (SDR) detector. However, we should note that the complexity of the proposed approach is less than that of those detectors. In the case of 8 antennas and 4 users, about 99% fewer computations are required when compared to the SDR and ML detectors.

INTRODUCTION

Multiple-input multiple-output (MIMO) systems divide a data stream into multiple unique streams, each of which is modulated and transmitted through a different radio-antenna chain at the same time in the same frequency channel. By taking advantage of multipaths, reflections of the signals, each MIMO receive antenna-radio chain is a linear combination of the multiple transmitted data streams. The data streams are separated at the receiver using MIMO algorithms that rely on estimates of all channels between each transmitter and each receiver. Hence, detection in MIMO systems is one of the fundamental problems.

The optimal model to minimize the joint probability error of detecting all the symbols simultaneously is the maximum likelihood (ML) detector [5]. It can be implemented using a brute-force search over all of the possible transmitted vectors or using more efficient search algorithms, for instance, the sphere decoder [**?**]. However, it has been shown that the expected computational complexity is exponential and impractical, for instance, there are 16^K vectors to be evaluated for K users with 16-QAM. Consequently, there has been much interest in implementing suboptimal detection algorithms. The most common suboptimal detectors are the linear receivers, i.e., the matched filter (MF), the decorrelator or zero forcing (ZF), the minimum mean-squared error (MMSE) detectors, decision feedback equalization (DFE), and the semidefinite relaxation (SDR) detector. There are many other suboptimal detection schemes ranging from lattice-based algorithms, alternating variable methods, to expectation maximization and many more [5].

The ZF algorithm is a straightforward approach to signal detection. The receiver with the ZF detector uses the estimated channel matrix to detect the transmitted signal as follows:

$$\tilde{\mathbf{s}} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{y} = \mathbf{H}^+\mathbf{y},$$

where $\mathbf{H}^H$ and $\mathbf{H}^+$ denote the Hermitian conjugate and the pseudo inverse respectively. Then each element of $\tilde{\mathbf{s}}$ is moved to the nearest constellation point.

One of the promising suboptimal detection strategies is the SDR detector [6]. The main reason for the high computational complexity of the ML detector is due to the fact that it is a non-convex combinatorial optimization problem. The approach of SDR is to formulate the ML problem in a higher dimension and then relax the non-convex constraints. Even though the SDR method solves the ML problem in polynomial time, it actually is not really practical for 16-QAM.

We propose a suboptimal detection algorithm, box constrained optimization which is much faster then the existing algorithms for ML problem. We apply the proposed algorithm for detection of 16-QAM signaling in MIMO system and compare it with the ZF detector and the SDR detector in [6].

16-QAM ML DETECTION AND BOX OPTIMIZATION

Consider the standard MIMO channel

$$\overline{\mathbf{y}} = \overline{\mathbf{H}}\overline{\mathbf{s}} + \overline{\mathbf{w}} \tag{1}$$

where $\overline{\mathbf{y}}$ is the received signal of length N, $\overline{\mathbf{H}}$ is an $N \times K$ channel matrix, $\overline{\mathbf{s}}$ is the length K vector of transmitted symbols, and $\overline{\mathbf{w}}$ is a length N complex random vector with normal distribution of zero mean and covariance $\sigma^2\mathbf{I}$. The symbols of $\overline{\mathbf{s}}$ belongs to some known complex constellation. In this paper, we consider a 16-QAM constellation, i.e., the real part and the imaginary part of $\overline{\mathbf{s}}_i$ for $i = 1, \ldots, K$ belong to the set $\{\pm 1, \pm 3\}$.

In order to avoid handling complex-valued variables, it is more convenient to use the following decoupled model:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w} \tag{2}$$

where

$$\mathbf{y} = \begin{bmatrix} Re\{\overline{\mathbf{y}}\} \\ Im\{\overline{\mathbf{y}}\} \end{bmatrix}; \mathbf{s} = \begin{bmatrix} Re\{\overline{\mathbf{s}}\} \\ Im\{\overline{\mathbf{s}}\} \end{bmatrix}; \mathbf{w} = \begin{bmatrix} Re\{\overline{\mathbf{w}}\} \\ Im\{\overline{\mathbf{w}}\} \end{bmatrix}; \mathbf{H} = \begin{bmatrix} Re\{\overline{\mathbf{H}}\} & -Im\{\overline{\mathbf{H}}\} \\ Im\{\overline{\mathbf{H}}\} & Re\{\overline{\mathbf{H}}\} \end{bmatrix}$$

Using these definitions, we can formulate the ML-detector of the transmitted symbols as

$$\text{ML:} \begin{cases} \min \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 \\ \text{subject to} \quad s_i \in \{\pm 1, \pm 3\},\ i = 1, \ldots, 2K. \end{cases} \tag{3}$$

The ML detector is a combinatorial problem and can be solved in a brute-force fashion by searching over all the $4^{2K} = 16^K$ possibilities. Clearly, as K increases, the brute-force search becomes prohibitively expensive.

As an alternative to a brute-force search, we compute an initial approximation to a solution of ML by solving the following continuous (relaxed) box-constrained optimization problem:

$$\text{RML:} \begin{cases} \min \|\mathbf{y} - \mathbf{Hs}\|^2 \\ \text{subject to } \; -3 \le s_i \le 3, \; i = 1, \ldots, 2K. \end{cases} \tag{4}$$

In RML, we ignore the integer constraints in ML and only require that x_i lies between -3 and $+3$. Let $\mathbf{A} = \mathbf{H}^T\mathbf{H}$, $\mathbf{b} = \mathbf{H}^T\mathbf{y}$, and $\mathbf{x} = \mathbf{s}$. Then, the relaxation of the ML problem RML is equivalent to the following quadratic programming problem with box constraints,

$$\begin{aligned} &\min \;\; \tfrac{1}{2}\mathbf{x}^T\mathbf{A}\mathbf{x} - \mathbf{b}^T\mathbf{x} \\ &\text{subject to } \;\; -3 \le x_i \le 3, \;\; i = 1, \ldots, 2K. \end{aligned} \tag{5}$$

SIMULATION RESULTS

In this section, we would like to compare the solutions obtained by our approach with those obtained by other strategies. In our first simulation, we considered a MIMO system with $K = 6$ inputs and $N = 12$ outputs using 16-QAM signaling. The entries of the MIMO channel were chosen as independent and identically distributed, zero-mean, complex normal random variables. For each signal to noise ratio (SNR), we use 150 channels and 10000 symbol sets for each channel to estimate the average probability of the error in detecting the message vectors. The box relaxation problem was solved by the AS_CBB method [3]. For comparison, we also simulated the conventional linear ZF detector and the SDR detector in [6]. The results are provided in Fig. 1 and 2.

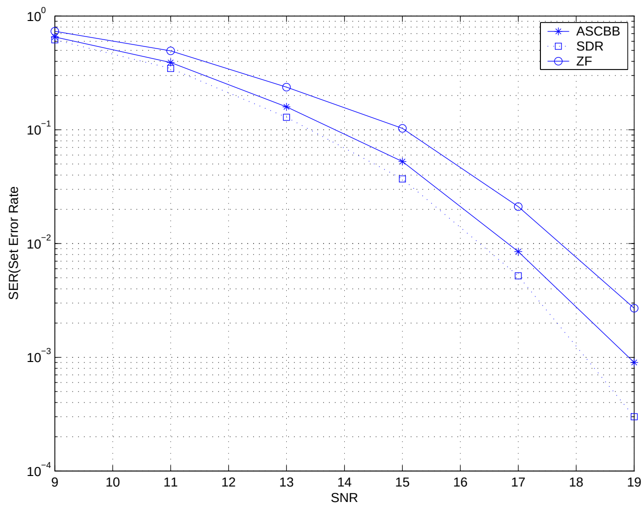

Figure 1. 12 antennas and 6 users

In the first simulation, we did not compare the performance of the box relaxation with that of the ML detector since the complexity of the ML detector is too high (16^6 possible vectors). In our second simulation, we consider a smaller system where a full ML search is impractical but possible, $K = 4$ and $N = 8$, i.e., 16^4 vectors. The rest of the parameters are as in the first simulation. The results are shown in Fig. 2. The gain of the proposed algorithm is about 0.9 dB and 2.3 dB over SDR and ZF at an set error rate (SER) of 10-2 in Figs. 1 and 2, respectively.

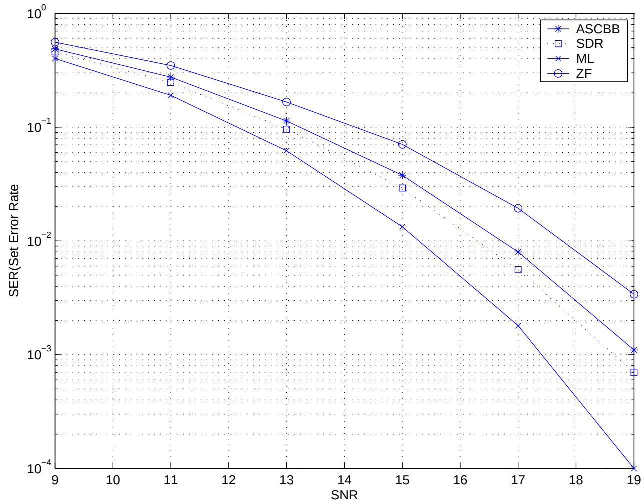

Figure 2. 8 antennas and 4 users

The loss of proposed algorithm is only about 0.1 dB over the ML. So the proposed algorithm is very effective with greatly reduced complexity.

Here, let us consider the complexities of the SDR, ZF and box relaxation detectors. The complexity of the SDR method in [6] is $O(R^{6.5}L^{6.5})$ where $L = O(K)$, K is the maximum of the number of symbols and the number of antennas, and R is the square root of the constellation [4]. It is easy to notice that the complexity of SDR depends not only the number of users and antennas, but also the order of the QAM. Since the ZF algorithm multiplies the received signal $\mathbf{y}$ by the pseudo inverse of the channel matrix $\mathbf{H}^+$, the complexity of the ZF detector is $O(L^3)$. The complexity of the box relaxation method is only $O(MNK)$, where M is maximum number of iterations, $M := O(\max\{-\log(\epsilon/\|\mathbf{x}_0 - \mathbf{x}^*\|), 0\}$ for its global minimum $\mathbf{x}^*$. In our computer simulation, the computation of the solution of the box relaxed approximation is about 100 times faster than solution time for the SDP method in [6], and even faster than the ZF detector, as expected by the complexity analysis.

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