OVERSAMPLING EXPANSION IN WAVELET SUBSPACES

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ABSTRACT

We find necessary and sufficient conditions for (shifted) oversampling expansions to hold in wavelet subspaces. In particular, we characterize scaling functions with the (shifted) oversampling property. We also obtain L^2 and L^∞ norm estimates for the truncation and aliasing errors of the oversampling expansion.

INTRODUCTION

Let $\{V_j\}_{j\in\mathbb{Z}}$ be an MRA (multi resolution analysis) generated by a scaling function $\phi(t)$, of which $\{\phi(t-n) \,:\, n\in\mathbb{Z}\}$ is a Riesz basis of V_0. As a natural extension of the classical Shannon sampling theorem on the Paley-Wiener spaces (cf. [6]), many authors considered the sampling theory on the wavelet subspace V_0 under various assumptions on the scaling function $\phi(t)$. For example, when $\phi(t)$ is a real-valued continuous orthonormal scaling function with decaying property $\phi(t) = O((1+|t|)^{-s})$, $s>1$, Walter [10] showed: if $\hat{\phi}^*(\xi) := \sum_{n\in\mathbb{Z}} \phi(n)e^{-in\xi} \neq 0$, then there is a Riesz basis $\{S(t-n) \,:\, n\in\mathbb{Z}\}$ of V_0 for which the regular sampling expansion

$$f(t) \;=\; \sum_{n\in\mathbb{Z}} f(n)S(t-n), \qquad f\in V_0 \tag{1}$$

holds. Walter's result has been extended further into the sampling theory on the shift invariant space $V(\phi) := \overline{\text{span}}\,\{\phi(t-n) \,:\, n\in\mathbb{Z}\}$ ([1,2,5,7–9,14]) under various conditions on the generator $\phi(t)$. On the other hand, Xia-Zhang [13] and Xia [12] (see also [4,11]) studied the oversampling property with rate $N(\geq 0)$ on V_0:

$$f(t) \;=\; \sum_{n\in\mathbb{Z}} f\left(\frac{n}{2^N}\right)\phi\left(2^N t - n\right), \qquad f\in V_0. \tag{2}$$

In order to relax rather strong constraints imposed on the scaling function $\phi(t)$ when we require either the regular sampling expansion (1) or the oversampling property (2), Chen and Itoh [3] considered the problem of oversampling expansion on V_0: Given an integer $N\geq 1$, when is there $S(t)\in V_0$ for which the oversampling expansion

$$f(t) \;=\; \sum_{n\in\mathbb{Z}} f\left(\frac{n}{2^N}\right) S\left(2^N t - n\right), \qquad f\in V_0 \tag{3}$$

holds on V_0? Assuming only $\{\phi(n)\}_{n\in\mathbb{Z}} \in l^2$, Chen and Itoh found a necessary and sufficient condition (see Theorem 1 in [3]) for the oversampling expansion (3) to hold on V_0 and estimated L^2 norms of the truncation and aliasing errors of the oversampling series thus obtained. However authors were not precise enough for necessary conditions on $\phi(t)$ to draw conclusions and there are some gaps in the arguments leading to main results in [3]. For example, the sample values $\phi(n)$ or $f\left(\frac{n}{2^N}\right)$ for functions $f(t) \in V_0$ and $n \in \mathbb{Z}$ may not be well defined unless we have suitable restrictions on the scaling function $\phi(t)$ since functions in V_0 are, in general, defined only a.e. in $\mathbb{R}$.

In this work, we first correct and extend the results on the oversampling expansion in [3] and then estimate both L^2 and L^∞ norms of the associated truncation and aliasing errors. We also characterize completely scaling functions having the oversampling property and the shifted oversampling property, which extends results in [11,12].

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