MICROMAGNETIC SIMULATIONS WITH LANDAU-LIFSHITZ-GILBERT EQUATION

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ABSTRACT

In a ferromagnetic material, the relaxation of the magnetization distribution is described by Landau-Lifshitz-Gilbert (LLG) equation. In this study, the LLG equation is numerically solved with the Successive Over Relaxation method, which has the enhanced stability and accuracy compared to the Gauss-Seidel approaches. The numerical example for an 1-dimensional test problem taken from Ref. 1 and the formulation for 2-dimensional equations are presented.

INTRODUCTION

In the recent days, the dynamics of micromagnetism [2] is one of the active research fields because of many applications such as magnetic memory devices. As a result, the theoretical study based on numerical simulation of Landau-Lifshitz-Gilbert (LLG) equation plays an important roles [3]. In general, the micromagnetic simulations have some difficulties due to a small time scale of pico-second level [1,4]. Therefore, advanced numerical techniques are required. The normalized LLG equation can be written as [2],

$$\mathbf{M}_t = -\gamma \mathbf{M} \times H - \frac{\gamma\alpha}{M_s}\mathbf{M} \times (\mathbf{M} \times H), \tag{1}$$

where γ is the gyromagnetic ratio, $M_s = |\mathbf{M}|$ is the saturation magnetization, α is the dimensionless damping coefficient, and *H* is the local field, derived from the free energy in the ferromagnet. On the right-hand side, the first and the second term denote the gyromagnetic and the damping term, respectively. In this abstract, we examined on the 1-dimensional (1-D) and the 2-dimensional (2-D) cases without damping term.

NUMERICAL METHODS AND RESULTS

The projection method for LLG equation, using the Gauss-Seidel approach, was shown to be unconditionally stable and more efficient than other numerical schemes [1,4]. However, in order to improve the convergence, we adopted the Successive Over Relaxation (SOR) method using implicit scheme [6,7]. As a simple test problem, we considered the 1-D LLG equation with $H = \partial^2\vec{m}/\partial x^2$ on $0 \le x \le 1$,

$$\frac{\partial \vec{m}}{\partial t} = -\vec{m} \times \frac{\partial^2 \vec{m}}{\partial x^2} + \vec{f} \tag{2}$$

with Neumann boundary condition

$$\frac{\partial \vec{m}}{\partial x}|_{x=0} = \frac{\partial \vec{m}}{\partial x}|_{x=1} = 0, \tag{3}$$

where the exact solutions are given by

$$m_x^{exact} = cos(x^2(1-x)^2)sin(t), \tag{4}$$

$$m_y^{exact} = sin(x^2(1-x)^2)sin(t), \tag{5}$$

$$m_z^{exact} = cos(t), \tag{6}$$

and

$$\vec{f} = \frac{\partial \vec{m}^{exact}}{\partial t} + \vec{m}^{exact} \times \frac{\partial^2 \vec{m}^{exact}}{\partial x^2}. \tag{7}$$

Then, the 1-D LLG equation can be discretized into

$$\begin{pmatrix} m_x^{n+1} \\ m_y^{n+1} \\ m_z^{n+1} \end{pmatrix} = \begin{pmatrix} m_x^n + \omega(g_x^n m_z^n - g_z^n m_y^n) \\ m_y^n + \omega(g_z^n m_x^{n+1} - g_x^{n+1} m_z^n) \\ m_z^n + \omega(g_x^{n+1} m_y^{n+1} - g_y^{n+1} m_x^{n+1}) \end{pmatrix} + \Delta t \begin{pmatrix} f_x^n \\ f_y^n \\ f_z^n \end{pmatrix}, \tag{8}$$

where

$$(I - \Delta t \Delta_h) g_i^n = m_i^n, i = x, y, z, \tag{9}$$

ω is a weighted factor between 1 and 2 and Δ_h is a finite difference approximation of Laplace operator.

Figure 1 shows numerical results of 1-D equation. Figure 1 (a), (b) and (c) display the magnetization distribution in each direction (solid black line: numerical results, dash red line:

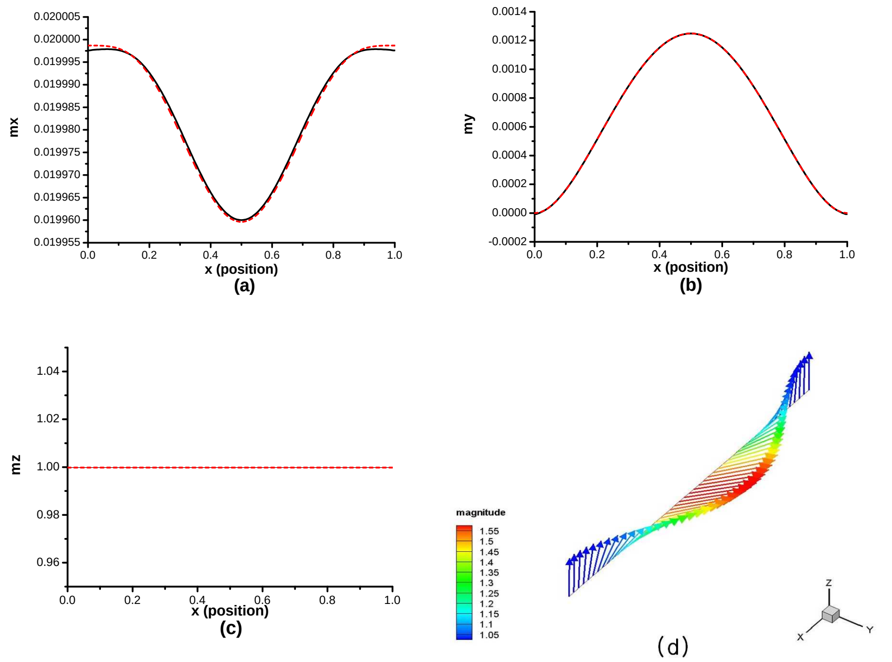

Figure 1. Magnetization distribution for 1-D LLG equation: (a) m_x, (b) m_y, (c) m_z, and (d) $\vec{m}$

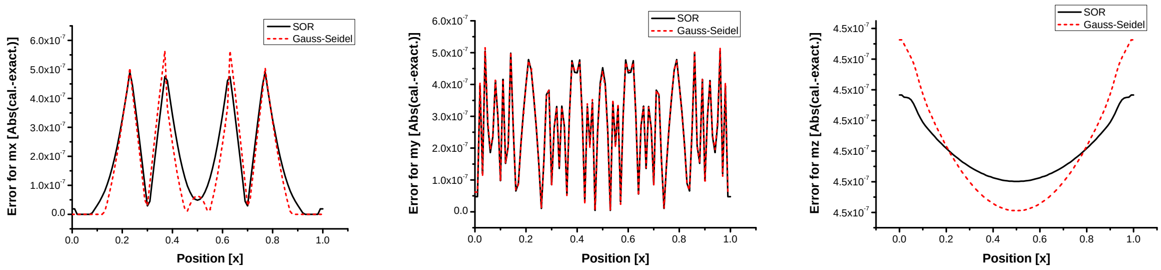

Figure 2. Absolute errors of the SOR method and the Gauss-Seidel method

exact solutions), calculated at $T = 0.02$ ($itr = 100$) and numerical results agree very well with exact solutions in the stripe geometry. Figure 1(d) is a vector sum in three dimension. However, its magnitudes are scaled to show the variation of vector field. Also we computed absolute errors between the SOR method and Gauss-Seidel method. Figure 2 shows absolute errors calculated by SOR method less than that in the Gauss-Seidel case.

Similarly, we considered 2-D LLG equation on $0 \leq x, y \leq 1$ as follows,

$$\frac{\partial \vec{m}}{\partial t} = -\vec{m} \times \left(\frac{\partial^2 \vec{m}}{\partial x^2} + \frac{\partial^2 \vec{m}}{\partial y^2}\right) + \vec{f} \tag{10}$$

with Neumann boundary condition

$$\frac{\partial \vec{m}}{\partial x}|_{x=0} = \frac{\partial \vec{m}}{\partial x}|_{x=1} = \frac{\partial \vec{m}}{\partial y}|_{y=0} = \frac{\partial \vec{m}}{\partial y}|_{y=1} = 0. \tag{11}$$

Then, 2-D LLG equation can be discretized as,

$$m^{n+1}_{x(i,j)} = m^n_{x(i,j)} + \omega\Delta t\{(\frac{m^n_{y(i+1,j)} - 2m^n_{y(i,j)} + m^n_{y(i-1,j)}}{(\Delta x)^2} + \frac{m^n_{y(i,j+1)} - 2m^n_{y(i,j)} + m^n_{y(i,j-1)}}{(\Delta y)^2})m^n_{z(i,j)}$$
$$-(\frac{m^n_{z(i+1,j)} - 2m^n_{z(i,j)} + m^n_{z(i-1,j)}}{(\Delta x)^2} + \frac{m^n_{z(i,j+1)} - 2m^n_{z(i,j)} + m^n_{z(i,j-1)}}{(\Delta y)^2})m^n_{y(i,j)}\},$$

$$m^{n+1}_{y(i,j)} = m^n_{y(i,j)} + \omega\Delta t\{(\frac{m^n_{z(i+1,j)} - 2m^n_{z(i,j)} + m^n_{z(i-1,j)}}{(\Delta x)^2} + \frac{m^n_{z(i,j+1)} - 2m^n_{z(i,j)} + m^n_{z(i,j-1)}}{(\Delta y)^2})m^{n+1}_{x(i,j)}$$
$$-(\frac{m^{n+1}_{x(i+1,j)} - 2m^{n+1}_{x(i,j)} + m^{n+1}_{x(i-1,j)}}{(\Delta x)^2} + \frac{m^{n+1}_{x(i,j+1)} - 2m^{n+1}_{x(i,j)} + m^{n+1}_{x(i,j-1)}}{(\Delta y)^2})m^n_{z(i,j)}\},$$

$$m^{n+1}_{z(i,j)} = m^n_{z(i,j)} + \omega\Delta t\{(\frac{m^{n+1}_{x(i+1,j)} - 2m^{n+1}_{x(i,j)} + m^{n+1}_{x(i-1,j)}}{(\Delta x)^2} + \frac{m^{n+1}_{x(i,j+1)} - 2m^{n+1}_{x(i,j)} + m^{n+1}_{x(i,j-1)}}{(\Delta y)^2})m^{n+1}_{y(i,j)}$$
$$-(\frac{m^{n+1}_{y(i+1,j)} - 2m^{n+1}_{y(i,j)} + m^{n+1}_{y(i-1,j)}}{(\Delta x)^2} + \frac{m^{n+1}_{y(i,j+1)} - 2m^{n+1}_{y(i,j)} + m^{n+1}_{y(i,j-1)}}{(\Delta y)^2})m^{n+1}_{x(i,j)}\},$$

where (i, j) represents a position (x_i, y_i), where $x_i = i\Delta x$, $y_i = j\Delta y$, $\Delta x = \frac{1}{N}$, $\Delta y = \frac{1}{M}$, N and M are discrete steps.

In summary, we compared the SOR method with the Gauss-Seidel one for the simple 1-D LLG equation and presented the formulation for 2-D problem. The SOR method shows the promising results for the more complicated problem. The numerical test on the 2-D case is still in the progress.

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REFERENCES

1. Wang, X. P., García-Cervera, C. J., and E, W., "A Gauss-Seidel projection method for micromagnetics simulations", *J. Comput. Phys.*, Vol. 171, 2001, pp. 357-372.
2. Brown Jr., W. F., *Micromagnetics, Interscience Tracts on Physics and Astronomy*, Willey-Interscience, New York/London, 1963.
3. Bolte, M. B. W., Möller, D. P. F., Meier, G. D., and Thieme, A., "Simulation of micromagnetic phenomena", *Proc. of 18th European Simulation Multiconference*, pp 407-441, Magdeburg Germany, June 2004.
4. E, W. and Wang, X. P., "Numerical methods for the Landau-Lifshitz equation", *SIAM J. Numer. Anal.*, Vol. 38, 2001, pp. 1647-1665.
5. Banas, L., "Numerical methods for the Landau-Lifshitz-Gilbert equation", *Lecture Notes in Computer Science*, Vol. 3401, 2005, pp. 158-165.
6. Burden, R. L. and Faires, J. D., *Numerical Analysis, Seventh edition*, Springer, Berlin, 1998.
7. Nakatani, Y., Uesake, Y., and Hayashi, N., "Direct solution of the Landau-Lifshitz-Gilbert equation for micromagnetics", *Jpn. J. Appl. Phys.*, Vol. 28, 1989, pp. 2485-2507.