Numerical Computational Method for Wood Drying

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ABSTRACT

A mathematical modeling for the drying process of hygroscopic porous media, such as wood, has been developed in the past decades. The governing equations for wood drying consist of three conservation equations with respect to the three state variables, moisture content, temperature and air density. They are involving simultaneous, highly coupled heat and mass transfer phenomena. In recent, the equations were extended to account for material heterogeneity through the density of the wood and via the density variation of the material process, capillary pressure, absolute permeability, bound liquid diffusivity and effective thermal conductivity. In this paper, we calculate the three primary variables of the drying process in terms of control volume finite element method to the heterogeneous transport model on unstructured grid.

MATHEMATICAL MODELLING

We introduced the mathematical formulation which represents the process of the wood drying, based on the model developed by Whitaker [7] and later by Perré and Turner [3]. The equations were extended to account for material heterogeneity through the density of the wood and via density variation of the material properties, capillary pressure, absolute permeability, bound liquid diffusivity and effective thermal conductivity.

The system of the three governing equations is summarized as follows:

Liquid Mass Conservation:

$$\frac{\partial}{\partial t}(\rho_0 X + \varepsilon_{\rm g}\rho_{\rm v}) + \nabla\cdot(\rho_{\rm w}\bar{\mathbf{v}}_{\rm w} + \rho_{\rm v}\bar{\mathbf{v}}_{\rm g} - \rho_0\bar{\bar{\rm D}}_{\rm b}\nabla X_{\rm b}) = \nabla\cdot(\rho_{\rm g}\bar{\bar{\rm D}}_{\rm v}\nabla w_{\rm v}),$$

Energy Conservation:

$$\frac{\partial}{\partial t}\Big(\rho_0(Xh_{\rm w}+h_{\rm s}) + \varepsilon_{\rm g}(\rho_{\rm v}h_{\rm v}+\rho_{\rm a}h_{\rm a}) - \int_0^{\rho_0 X_{\rm b}}\Delta h_{\rm w}d\rho - \varepsilon_{\rm g}P_{\rm g}\Big)$$

$$+\nabla\cdot(\rho_{\rm w}h_{\rm w}\bar{\mathbf{v}}_{\rm w} + (\rho_{\rm v}h_{\rm v}+\rho_{\rm a}h_{\rm a})\bar{\mathbf{v}}_{\rm g} - h_{\rm b}\rho_0\bar{\bar{\rm D}}_{\rm b}\nabla X_{\rm b})$$
$$= \nabla\cdot(\rho_{\rm g}\bar{\bar{\rm D}}_{\rm v}(h_{\rm v}\nabla w_{\rm v} + h_{\rm a}\nabla w_{\rm a}) + \bar{\bar{\rm K}}_{\rm eff}\nabla T),$$

Air Conservation:

$$\frac{\partial}{\partial t}(\varepsilon_{\rm g}\rho_{\rm a}) + \nabla\cdot(\rho_{\rm a}\bar{\mathbf{v}}_{\rm g}) = \nabla\cdot(\rho_{\rm g}\bar{\bar{\rm D}}_{\rm v}\nabla w_{\rm a}).$$

The primary variables in system are the moisture content X, the temperature T and the intrinsic phase air density $\bar{\rho}_{\mathrm{a}} = \varepsilon_{\mathrm{g}}\rho_{\mathrm{a}}$. The remaining symbols are secondary variables and parameters. The transport coefficients that are necessary for numerical computation were referred to Truscott and Turner [6].

The boundary conditions are classified two types. One is external boundaries and the other is symmetry boundaries. The boundary conditions proposed for the external drying surfaces are given as

$$\mathbf{J}_{\mathrm{w}} \cdot \hat{\mathbf{n}} = k_{\mathrm{m}} c M_{\mathrm{v}} \ln\left(\frac{1 - x_{\mathrm{v}\infty}}{1 - x_{\mathrm{v}}}\right),$$

$$\mathbf{J}_{\mathrm{e}} \cdot \hat{\mathbf{n}} = q(T - T_{\infty}) + h_{\mathrm{v}} k_{\mathrm{m}} c M_{\mathrm{v}} \ln\left(\frac{1 - x_{\mathrm{v}\infty}}{1 - x_{\mathrm{v}}}\right),$$

$$P_{\mathrm{g}} = P_{\infty},$$

where $\mathbf{J}_{\mathrm{w}}$ and $\mathbf{J}_{\mathrm{e}}$ represent the fluxes of liquid and energy at the boundary, respectively, $\hat{\mathbf{n}}$ is the outward unit normal vector, h_{v} and k_{m} the heat and mass transfer coefficients, respectively, x_{v} and $x_{\mathrm{v}\infty}$ the molar fractions of vapor at the exchange surface and in the air, respectively, and c the molar concentration. The pressure at the external drying surface is given at the atmospheric pressure. At symmetric planes, all fluxes of liquid, heat, vapor and air are set to zero.

Also, we need some of initial conditions. The initial average moisture content $\bar{X}$ and temperature T are given by 120% and $25^{\circ}\mathrm{C}$, respectively. Then initial moisture content distribution has to be determined prior to the commencement of the drying process. The liquid saturation $S_{\mathrm{w}i}(, i = 1, \ldots, N)$ at each node and the equilibrium capillary pressure $P_{c\mathrm{eqm}}$ are calculated from the nonlinear system of $(N + 1)$ equations as follows:

$$P_c(S_{\mathrm{w}i}, \rho_{\mathrm{o}i}, T) = P_{c\mathrm{eqm}}, \quad i = 1, \ldots, N$$

$$\rho_{\mathrm{w}} \frac{\sum_i \phi_i S_{\mathrm{w}i} A_i}{\sum_i \rho_{\mathrm{o}i} A_i} + X_{\mathrm{fsp}} = \bar{X},$$

where A_i is the area corresponding to the node i. Once the values of $S_{\mathrm{w}i}$ have been determined using Newton iteration, the initial moisture content can be calculated using:

$$X_i = \frac{\rho_{\mathrm{w}} \phi_i S_{\mathrm{w}i}}{\rho_{\mathrm{o}i}} + X_{\mathrm{fsp}}, \quad i = 1, \ldots, N$$

For the computation of drying process, we also need a suitable drying schedule. The dry and wet bulb temperatures were ramped up to their kiln operating values of $60^{\circ}\mathrm{C}$ and $40^{\circ}\mathrm{C}$, respectively, over a period of 10 minutes.

NUMERICAL SOLUTION

We find the numerical solution by solving the transport equations in terms of control volume finite element methods. [1] [2] [6]

The discretized formulation with boundary conditions is highly coupled system of nonlinear equations. Then we use the inexact Newton iterative method to solve this system. Also we use the Bi-CGSTAB method to solve linear system.

A numerical simulation for the process of the wood drying has some of difficulties, for instance, high aspect ratios of domain, $1 : 10$ or $1 : 50$, high anisotropy ratios (of the order of 1000 for permeability), tightly coupled equations, highly non-linear equations, non-linear

boundary conditions, steep moisture and pressure gradients, highly convective internal gaseous flows and outer and inner iteration stages make another stumbling block

In this study we find a appropriate method, so called control volume finite element method, which is convenient to non-dimensionalise the system of equations, and be able to utilize an unstructured mesh for processing. Also it has a number of different alternatives for the exact shape of CV and is flexible for evaluating fluxes through faces

A complete CVFE method is suitable for resolving nonlinear transport equations on triangular meshes

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