ITERATIVE CORRECTION FOR SPECT IMAGE DISTORTED BY COLLIMATOR'S CHARACTERISTIC

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ABSTRACT

The projection data of the emission activity of interests is blurred by the point response function of the collimator that is used to define the range of directions where photons can be detected. The point response function of the collimator is not spatially invariant, but it depends on the distance between the point and the collimator. Conventional Fourier correction methods limit deblurring ability due to the simplification of the Fourier description, and are weak against to noise, since the deblurring process follows an inverse filtering. In this work we use ML-EM correction method to reconstruct images with efficient removal of the distance dependent blurring. We compute the projector and the backprojector as a sum of convolutions with distance-dependent point response functions instead of matrix form. We conducted several simulation studies to compare the performance of the proposed method with that of Fourier correction methods. The result shows that the proposed method outperforms Fourier correction methods objectively and subjectively.

APERTURE CORRECTION

The purpose of collimation in SPECT is to allow photons from only a limited range of directions to be detected. As compared with ideal collimation, which can be achieved by using a long parallel hole with a very small size, nonideal collimation is practical, but causes spatially variant blurring in observed projection data and hence in reconstructed images. The main objective of this work is to present an efficient de-blurring method to obtain better resolution in reconstructed images in the presence of noise in projection data.

The imaging geometry in two-dimensional plane is shown in Figure 1-a. Here we use following notations:

- (s, t) rotated rectangular coordinates by θ from (x, y) coordinates
- $Y(\theta, u)$ projection of $f(x, y)$ by ideal collimation at (θ, u)
- $Z(\theta, u)$ projection of $f(x, y)$ by nonideal collimation at (θ, u)

Here the ideal collimation means that the aperture angle in collimation is zero and hence collimation itself becomes the integral over a straight line:

$$Y(\theta, u) = \int_t \int_s f(s\cos\theta - t\sin\theta, s\sin\theta + t\cos\theta)\delta(u - s)\, ds\, dt, \tag{1}$$

where δ is the Dirac delta function. To maintain the sensitivity, however, we need to have a non-zero aperture angle, and thus the region to be detected by parallel hole is not straight beam. See Figure 1-b.

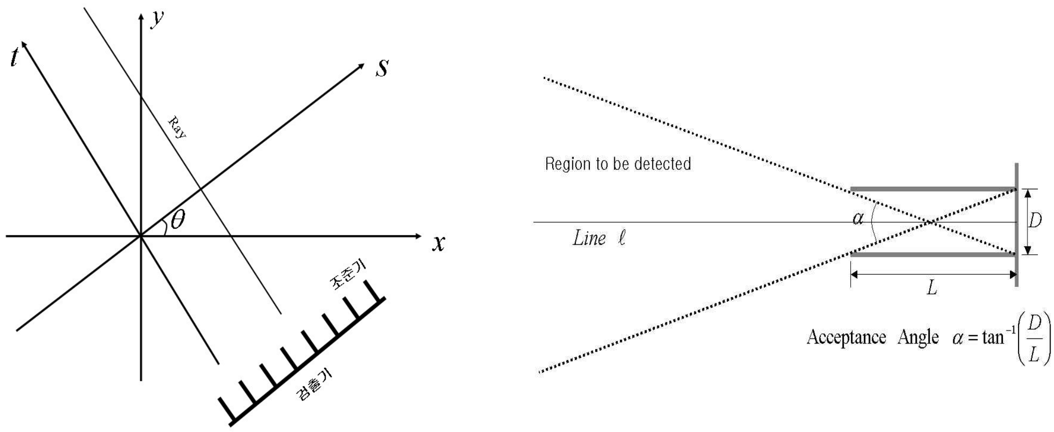

Figure 1. a(left): Imaging geometry, b(right): Acceptance angle

To consider this kind of phenomenon, Xia et. al. [1] models the sinogram by nonideal collimation as

$$Z(\theta, u) = \int_t \int_s f(s\cos\theta - t\sin\theta, s\sin\theta + t\cos\theta) h(u - s, t + R)\, ds\, dt, \tag{2}$$

where h is the PRF(Point Response Function). Let $\hat{Y}(\theta, \omega)$ be the Fourier transform of $Y(\theta, u)$ as a function of u, and $\hat{\hat{Y}}(n, \omega)$ be the Fourier coefficients of $\hat{Y}(\theta, \omega)$ as a function of θ, and similarly for $\hat{Z}(\theta, \omega)$ and $\hat{\hat{Z}}(n, \omega)$. Thus for a point source at $(r_p \cos\phi_p, r_p \sin\phi_p)$ in (x, y) co-ordinates, we have

$$\hat{\hat{Y}}(n, \omega) = \frac{1}{2\pi} \int_0^{2\pi} e^{-2\pi i(\omega s_p + n\theta)}\, d\theta \tag{3}$$

and

$$\hat{\hat{Z}}(n, \omega) = \frac{1}{2\pi} \int_0^{2\pi} \hat{h}(\omega, t_p + R) e^{-2\pi i(\omega s_p + n\theta)}\, d\theta, \tag{4}$$

where $s_p = r_p \cos(\phi_p - \theta)$ and $t_p = r_p \sin(\phi_p - \theta)$ and $\hat{h}(\omega, t_p + R)$ is the Fourier transform of $h(s, t_p + R)$. As we can see in (3) and (4), without resorting to a simplification, it is very difficult to get an efficient algorithm. Xia et. al.[1] simplify the relation between $\hat{\hat{Y}}(n, \omega)$ and $\hat{\hat{Z}}(n, \omega)$ to

$$\hat{\hat{Y}}(n, \omega) \approx \frac{\hat{\hat{Z}}(n, \omega)}{\hat{h}(\omega, -\frac{n}{\omega} + R)} \tag{5}$$

by using the fact that the integration with a rapidly oscillating function is negligible. This Fourier correction method has mainly two drawbacks; first, the simplification used in (5) limits the de-blurring effect, and, second, the division in (5) is very unstable, since we must expect noisy sinogram for $Z(\theta, u)$ in practice.

PROPOSED METHOD

To deal with the collimator blurring, we assume a linear algebraic model[2] and use ML-EM iteration[3] for image reconstruction. We use induces following notations with induces b for projection bins and v for phantom image voxels:

f_v activity at voxel v

Z_b photon counts by nonideal collimation at projection bin b

$P_{b,v}$ entries of the system matrix P

Using these notations, we can write the relation between unknown phantom image voxel values f_v and projection data Z_b as $Z_b = \sum_v P_{b,v} f_v$. Then the problem of de-blurring and reconstruction becomes finding a solution of the system of linear equations. Following this approach, we use ML-EM[3], where the new image f^{n+1} is updated from the current image f^n by

$$f_v^{n+1} = f_v^n \sum_b P_{b,v} \frac{Z_b}{\mu_b^n} \Big/ \sum_c P_{c,v}, \quad \mu_v^n = \sum_v P_{b,v} f_v^n. \tag{6}$$

The computation of matrix $P_{b,v}$ requires long computation time and huge data storage requirement. In [4], the rotational symmetry between the sinogram and the phantom image has been exploited. Notice that the equation (2) implies that the projection is the integral sum of convolutions of rotated phantom image f by angle θ and distance-dependent PRF $h(s, t+R)$. Therefore, by using continuous variables used in (2), we can accurately approximate the projection of (f_v), which is performed by $\sum_v P_{b,v} f_v$ in matrix-vector multiplication, by a sum of convolutions as

$$\sum_v P_{b,v} f_v \approx \sum_{(\theta,u)\in b} \int\int \mathbb{R}_\theta f(s,t) h(u-s, t+R)\, ds\, dt, \tag{7}$$

where $(\theta, u) \in b$ indicates the case when continuous projection variables (θ, u) belongs to the region occupied by the projection bin b and $\mathbb{R}_\theta f$ is the rotated version of f by θ. We compute the backprojection of (Z_b), which is performed by $\sum_b P_{b,v} Z_b$, by a sum of convolutions as

$$\sum_b P_{b,v} Z_b \approx \sum_{(x,y)\in v} \int \mathbb{R}_{-\theta} \left(\int Z(\theta, u) h(s-u, t+R)\, du \right) d\theta. \tag{8}$$

The major advantage of proposed iterative methods over Fourier correction methods[1] is the fact that the de-blurring process in the proposed method does not require a simplification, which often leads to an unsatisfactory result, and the division in the Fourier transform domain, which is the main reason of Fourier correction method's weakness against noise.

SIMULATION AND CONCLUSION

We conducted several computer simulation with 'hot rods' and 'cold rods' phantom images to compare the performance of the proposed method with that of one of Fourier correction method[1]. We assumed the acceptance angle to be $9.58°$ and generated projection data by using Monte Carlo simulations. Simulation results by Fourier correction method are shown in Figure 2. As we can see, the de-blurring effect by the Fourier correction method is rather poor, and reconstructed images are rather noisy even though we used 0.5 as the cut-off. These results indicate that the division in the frequency domain is still unstable, and produces noisy reconstructed images.

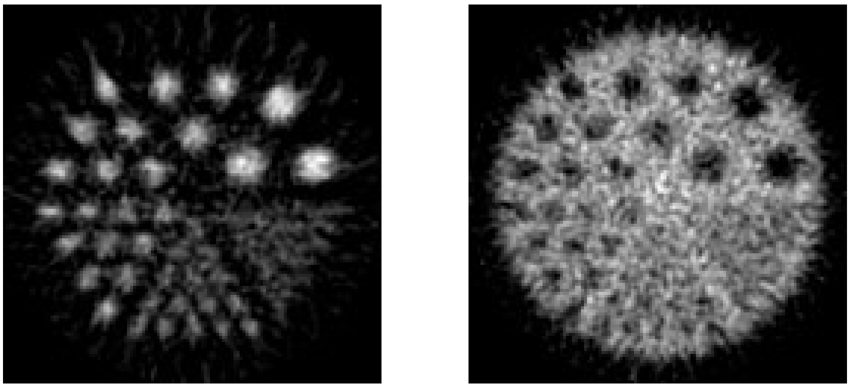

Figure 2. Reconstructed images by Fourier correction, a(left) - ‘hot rods’ with RMS = 0.278, b(right) - ‘cold rods’ with RMS = 0.117,

Simulation results by proposed method are shown in Figure 3. Visual comparison, even though it is rather subjective in nature, clearly shows that the proposed method provides a better de-blurring effect and less noisy reconstruction than the Fourier correction method. It is also true that RMS(Root-Mean-Square) errors by the proposed method are smaller than those by the Fourier correction method. See RMS values written in captions of Figures. Based on these subjective and objective results, we can conclude that the proposed method by a system-specific convolution-based projector/backprojector outperforms Fourier correction methods.

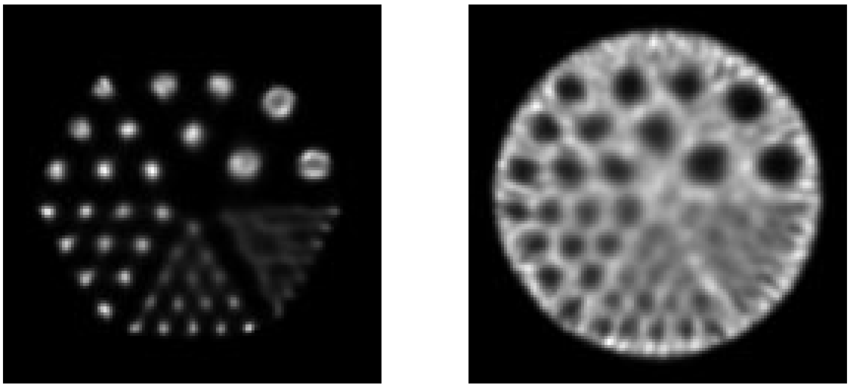

Figure 3. Reconstructed images by proposed method, a(left) - ‘hot rods’ with RMS = 0.236, b(right) - ‘cold rods’ with RMS = 0.082

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