A FRACTIONAL STEP MESHFREE POINT COLLOCATION METHOD FOR THE INCOMPRESSIBLE NAVIER-STOKES EQUATIONS

Yongsik Kim [1]

1) *Department of General Education, KookMin University, Seoul 136-702, KOREA*

Corresponding Author : Yongsik Kim, yskim67@kookmin.ac.kr

ABSTRACT

A fractional-step point collocation method by meshfree approximation is presented for solving non-stationary incompressible Navier-Stokes equations. A single node set and auxiliary local points to each node point are used in the present study. In the momentum equations, the diffusion terms are discretized by a meshfree approximation FMLSRK(Fast Moving Least Square reproducing Kernel Approximation) on a given node set, while the inertia terms are discretized by using local points. In the pressure correction step, the divergence of intermediate velocity is calculated using local points which results local mass conservation. The Crank-Nicolson method is employed to the viscous and the convective terms for the second order accuracy in time stepping. The present numerical scheme is applied to several numerical experiments to prove its accuracy and efficiency.

INTRODUCTION

From the pioneering work of Chorin[1], numerous numerical methods for the Navier-Stokes equations have been introduced which split the momentum equations(1) into two parts. They share the idea that the effect of the pressure gradient in the momentum equation could be recovered from a certain projection step which inherits the Hodge decomposition. The projection method[1,2] and the fractional-step method[3,4] are widely used names of these methods. There are variants of the projection method such as, the Gauze-Uzawa algorithm[5], the block-factorization[6], the quasi-compressibility scheme(also known as the Yosida method) and more.

In this paper, we propose a fractional-step point collocation scheme by FMLSRK for the non-stationary incompressible Navier-Stokes equations in $\mathbb{R}^n(n = 2, 3)$,

$$\mathbf{u}_t - \nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f}, \quad \text{in } \Omega \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0, \quad \text{in } \Omega \tag{2}$$

$$\mathbf{u} = \mathbf{g}, \quad \text{on } \partial\Omega \tag{3}$$

where $\mathbf{u}$ is the velocity vector, p is the pressure which is determined up to constant, $\mathbf{f}$ is the external force and ν is the fluid viscosity.

The meshfree point collocation method[7,8] is newly developed numerical scheme which has a strong point to the problems on a complicated geometry. Consider $\mathbf{u}^*$ and q^n are the intermediate velocity and the pseudo-pressure. $\mathbf{u}^n$ and p^n represent the velocity and the pressure at the n-th time step. The following is the proposed algorithm.

- **STEP 1**: Solve the intermediate velocity.

$$\frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} - \frac{\nu}{2}(\Delta \mathbf{u}^* + \Delta \mathbf{u}^n) + \frac{1}{2}\left(\mathbf{u}^n \cdot \widetilde{\nabla} \mathbf{u}^* + \mathbf{u}^* \cdot \nabla \mathbf{u}^n\right) = \mathbf{f}^n \qquad \text{in } \Omega, \tag{4}$$

$$\mathbf{u}^* = \mathbf{u}^{n+1} + \Delta t \frac{\partial q^n}{\partial \vec{\tau}} \vec{\tau} \qquad \text{on } \partial\Omega. \tag{5}$$

- **STEP 2**: Pressure correction.

$$\Delta q^{n+1} = \frac{1}{\Delta t} \widetilde{\nabla} \cdot \mathbf{u}^* \qquad \text{in } \Omega, \tag{6}$$

$$\frac{\partial q^{n+1}}{\partial \vec{\mathbf{n}}} = 0 \qquad \text{on } \partial\Omega. \tag{7}$$

- **STEP 3**: Update field variables. Go to step 1 or stop.

$$p^{n+1} = q^{n+1} - \nu \widetilde{\nabla} \cdot \mathbf{u}^*, \tag{8}$$

$$\mathbf{u}^{n+1} = \mathbf{u}^* - \Delta t \nabla q^{n+1}. \tag{9}$$

Here, $\mathbf{f}^n$ is the external force at n-th time step, Δt is the size of time step, $\vec{\tau}$ and $\vec{\mathbf{n}}$ are unit tangent vector and unit normal vector on $\partial\Omega$. In the algorithm, except $\widetilde{\nabla}$ in (4) and $\widetilde{\nabla}\cdot$ in (6)(8), all the differential operator are discretized by meshfree approximation on a given node set. The auxiliary local points will be used to discretize $\widetilde{\nabla}$ in (4) and $\widetilde{\nabla}\cdot$ in (6)(8).

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