ADAPTIVE MESH REFINEMENT FOR THE BLACK-SCHOLES EQUATIONS

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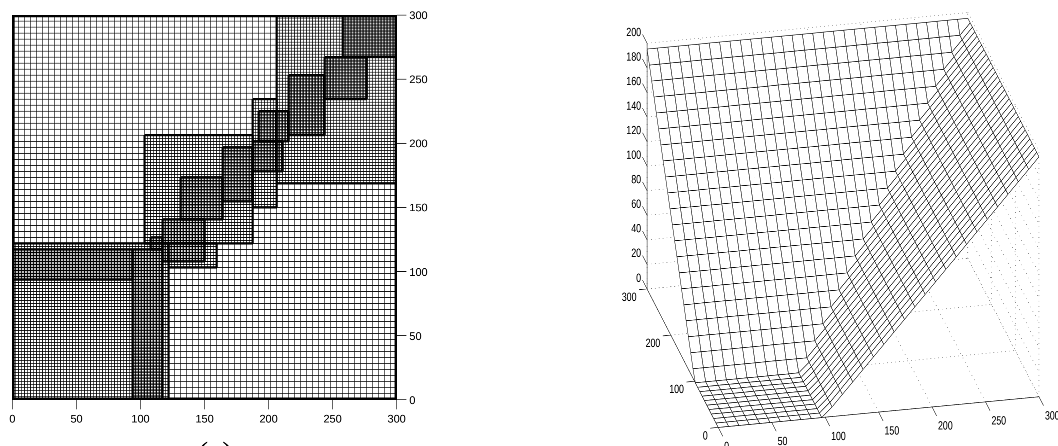
ABSTRACT

In this study, we present a time-dependent adaptive mesh refinement method for Black-Scholes equations in two dimensions. Finite difference methods for the solution of partial differential equations on block-structured adaptively refined rectangular grids are used.



(a)

(b)

Figure 1. (a) Initial mesh. (b) Option price.

After time reversion the Black-Scholes-PDE forms the initial value problem[2]:

$$\frac{\partial u}{\partial t} = \frac{1}{2}\sum_{i,j=1}^{n} \sigma_i \sigma_j \rho_{ij} S_i S_j \frac{\partial^2 u}{\partial S_i \partial S_j} + r\sum_{i=1}^{n} S_i \frac{\partial u}{\partial S_i} - ru = 0, \tag{1}$$

$$\text{for } (\mathbf{S}, t) \in \mathbf{R}_+^n \times (0, T),$$

$$u(\mathbf{S}, 0) = u_0(\mathbf{S}) = \left(K - \sum_{i=1}^{d} S_i\right)_+ \quad \text{for } \mathbf{S} \in \mathbf{R}_+^n. \tag{2}$$

In the two dimensions,

$$\frac{\partial u}{\partial t} = \frac{1}{2}(\sigma_1 S_1)^2 \frac{\partial^2 u}{\partial S_1^2} + \frac{1}{2}(\sigma_2 S_2)^2 \frac{\partial^2 u}{\partial S_2^2} + \sigma_1 \sigma_2 \rho S_1 S_2 \frac{\partial^2 u}{\partial S_1 \partial S_2} \tag{3}$$

$$+ rS_1 \frac{\partial u}{\partial S_1} + rS_2 \frac{\partial u}{\partial S_2} - ru = 0, \quad \text{for } (\mathbf{S}, t) \in \mathbf{R}_+^2 \times (0, T),$$

$$u(\mathbf{S},0) = u_0(\mathbf{S}) = \left(K - \sum_{i=1}^{2} S_i\right)_+ \text{ for } \mathbf{S} \in \mathbf{R}_+^2. \quad (4)$$

In this presentation, we will use a time-dependent adaptive mesh refinement method, which have been developed for thin film equations [3] and adaptive Cahn-Hilliard equations[4]. Fig. 1 shows the initial adaptive mesh structure and option price. Computational domain size $[0,300]\times[0,300]$, the effective mesh size 256×256.

$$\sigma_1 = 0.5, \sigma_2 = 0.5, \rho = 0.5, r = 0.03. \quad (5)$$

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