TAIL ASYMPTOTICS FOR THE WAITING TIME IN AN M/G/1 RETRIAL QUEUE

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ABSTRACT

We consider an M/G/1 retrial queue, where the service time distribution has a regularly varying tail with index $-\beta$, $1 < \beta < 2$. It is shown that the waiting time distribution has a regularly varying tail with index $1-\beta$, and the pre-factor is determined explicitly. The result is obtained by comparing the waiting time in the M/G/1 retrial queue with the waiting time in the ordinary M/G/1 queue with random order service policy.

INTRODUCTION

We consider an M/G/1 retrial queueing system, where customers arrive according to a Poisson process with intensity λ, service times for customers are independent and identically distributed with distribution function F_B, and there is a single server. If the server is idle at the time of a customer arrival, the arriving customer begins to be served immediately and leaves the system after service completion. Otherwise, i.e., if the server is busy, the arriving customer joins a retrial group, called an orbit. While in orbit, each customer spends an exponential time with mean ν^{-1} before visiting the server again. If an incoming repeated customer finds the server idle, it is served and leaves the system after service completion. Otherwise, i.e., if the repeated customer finds the server busy, the customers comes back to the orbit immediately, and tries his luck after an exponential time with mean ν^{-1} again. The traffic load ρ is defined as $\rho = \lambda \mathbb{E}B$, where $\mathbb{E}B$ denotes the mean service time. It is assumed that $\rho < 1$ for the stability of the system.

We assume that the service time distribution has a regularly varying tail with index $-\beta$, $1 < \beta < 2$, i.e.,

$$1 - F_B(x) \sim cx^{-\beta} L(x),$$

where $c > 0$, L is a slowly varying function and and $f(x) \sim g(x)$ denotes $\lim_{t\to\infty} \frac{f(x)}{g(x)} = 1$.

The main interest of this research is the tail behavior of the waiting time distribution in the retrial queue. We show that the waiting time distribution has a regularly varying tail with index $1-\beta$, and we also gives the pre-factor explicitly. The result is obtained by comparing the waiting time in the M/G/1 retrial queue with the waiting time in the ordinary M/G/1 queue with random order service policy.

MAIN RESULTS

Let

$$N(t) = \text{the number of customers in the orbit at } t;$$

$$C(t) = \begin{cases} 1, \text{ if the server is busy at } t, \\ 0, \text{ if the server is idle at } t; \end{cases}$$

$$X(t) = \begin{cases} \text{the elapsed service time of the customer who is in service at } t, & \text{if } C(t) = 1, \\ 0, & \text{if } C(t) = 0. \end{cases}$$

Then $\{(N(t), C(t), X(t) : t > 0\}$ is a Markov process. Let

$$\tau = \inf\{t > 0 : N(t) = 0, C(t) = 0\}$$

$$G(t) = \mathbb{P}(\tau \leq t \mid N(0) = 0, C(0) = 1, X(0) = 0\}$$

Theorem 1 *The distribution function G satisfies*

$$1 - G(x) \lesssim 1 - F_B(x),$$

where $f(x) \lesssim g(x)$ denotes $\lim_{x\to\infty} \frac{f(x)}{g(x)} < \infty$.

We consider the corresponding ordinary M/G/1 queue with random order service policy, where the arrival rate is λ and service times have distribution function F_B. Let W^{ROS} denote a generic random variable for the waiting time of an arbitrary customer.

Theorem 2 *The waiting time distribution in the M/G/1 retrial queue has the same tail asymptotics as the waiting time distribution in the corresponding ordinary M/G/1 queue with random order service policy, i.e.,*

$$\mathbb{P}(W > x) \sim \mathbb{P}(W^{ROS} > x).$$

Using the results of Boxma et al. (2004) on the regularly varying tail of the waiting time distribution in the M/G/1 queue with random order service, we obtain the following theorem:

Theorem 3 *The waiting time distribution in the M/G/1 retrial queue has the following tail asymptotics:*

$$\mathbb{P}(W > x) \sim cx^{1-\beta}L(x),$$

where

$$c = \frac{\rho}{1-\rho}h(\rho, \beta)\frac{1}{\beta - 1}\frac{1}{EB},$$

$$h(\rho, \beta) = \int_0^1 f(u, \rho, \beta)du,$$

$$f(u, \rho, \beta) = \frac{\rho}{1-\rho}(\frac{\rho u}{1-\rho})^{\beta-1}(1-u)^{\frac{1}{1-\rho}} + (1 + \frac{\rho u}{1-\rho})^{\beta}(1-u)^{\frac{1}{1-\rho}-1}.$$

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