A BDDC ALGORITHM FOR THREE DIMENSIONAL ELASTICITY WITH MORTAR DISCRETIZATION

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ABSTRACT

A BDDC (balancing domain decomposition by constraints) algorithm is developed for compressible elasticity problems in three dimensions with mortar discretization on geometrically nonconforming subdomain partitions. Material parameters of the elasticity problems may have jump across the subdomain interface. Coarse basis functions in the BDDC algorithm are constructed from primal constraints on faces, that are similar to the average matching condition and the moment matching condition considered in [6,3]. A condition number bound is proved to be $C(1+\log(H/h))^3$ for geometrically non-conforming partitions as well as to be $C(1+\log(H/h))^2$ for geometrically conforming partitions. The bound is not affected by the jump of the material parameters across the subdomain interface. Numerical results are included.

INTRODUCTION

This study is intended for development of an efficient BDDC algorithm for solving mortar discretization of elasticity problems. The mortar discretization is applied to geometrically non-conforming subdomain partitions. In the geometrically non-conforming partitions, a pair of subdomains can intersect over only part of a subdomain face or a subdomain edge. Allowing geometrically non-conforming partitions makes mortar discretizations much more practical for problems given on three dimensional complex structures with composite materials.

BDDC algorithms have been studied extensively in [10–12,8,9,4] since its introduction by Dohrmann [2]. Coarse component of BDDC preconditioners is built from weighted sum of functions that minimize discrete local energies subject to certain primal constraints across the subdomain interface. The BDDC algorithms solve linear systems of primal unknowns in contrast to FETI-DP algorithms, that have been applied successfully to solving algebraic systems of dual unknowns during the last decade [1,7,6,3]. This feature makes the BDDC algorithms more robust and more flexible than the FETI-DP algorithms; see [12,13]. On the other hand, the BDDC and FETI-DP algorithms have much in common. They share the same spectra except the eigenvalues 0 and 1 when the same set of primal constraints is chosen; see [11,8]

Both of these methods have been applied to solving problems arising from mortar discretizations. In a recent study by the author jointly with Dryja and Widlund [4], a BDDC algorithm is developed for elliptic problems. This algorithm was shown to share the same spectra with the FETI-DP algorithm considered in [5] by the author and Lee. In addition, it is generalized to the geometrically non-conforming partitions with a slightly weaker condition number bound, $C(1+\log(H_i/h_i))^3$. The FETI-DP algorithm in [5] has been extended to solving elasticity problems by the author [3]. However, the work is limited to geometrically conforming partitions. This study aims at developing a BDDC algorithm for the elasticity problems that allows the geometrically non-conforming partitions.

FETI-DP algorithms for compressible elasticity problems employing the exact matching condition have been developed by Klawonn and Widlund [6]. Even for the compressible case, extending FETI-DP algorithms to the three dimensional elasticity problems needed much more effort than the standard second order problems, i.e., elliptic problems. A special difficulty arises in building certain functionals dual to six rigid body motions. These functionals are important in providing a condition number bound independent of the number of subdomains. The most important ingredient of this study is a new set of primal constraints that makes it possible to construct such functionals in geometrically nonconforming partitions employing the mortar matching condition. Using the new set of the primal constraints, we build a scalable BDDC algorithm with its condition number bound, $C(1+\log(H_i/h_i))^3$.

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