SYMMETRIC TIGHT WAVELET FRAMES CONSTRUCTED FROM QUASI-INTERPOLATORY SUBDIVISION MASKS

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ABSTRACT

This talk presents tight wavelet frames with two compactly supported symmetric generators of more than one vanishing moments in Unitary Extension Principle. We determine all possible free tension parameters of the quasi-interpolatory subdivision masks whose corresponding refinable functions guarantee our wavelet frame. In order to reduce shift variance of the standard discrete wavelet transform, we use the three times oversampling filter bank and eventually obtain a ternary (low, middle, high) frequency scale. In applications to signal/image denoising and erasure recovery, the results demonstrate reduced shift variance and better performance of our wavelet frame than the usual wavelet systems such as Daubechies wavelets.

INTRODUCTION

As one constructs wavelet systems, the important characteristics for efficient approximation of functions are approximation orders and vanishing moments of wavelet systems. The B-splines and interpolatory refinable functions are suitable candidates for high approximation orders since they have highest approximation orders with short supports and the possibility to guarantee framelets with high vanishing moments. However, when it comes to construct symmetric tight wavelet frames with two generators from these refinable functions using Unitary Extension Principle, we cannot obtain framelets with high vanishing moments. For the B-splines, the vanishing moment of at least one of the frame generators is only one, and for the interpolatory refinable functions, only the piecewise linear B-spline is possible for construction of symmetric tight wavelet frames with two generators. Motivated by this result, we consider the quasi-interpolatory subdivision masks with a tension parameter in [3] as the lowpass filters. We study the quasi-interpolatory subdivision masks for the construction of anti- or symmetric compactly supported tight wavelet frames with essentially two frame generators using a three times oversampling filter bank as Figure 1. For three times oversampling framelet transform, we need three frame generators one of which is half integer shift version of another generator, which explains the meaning of 'essentially two frame generators'. Because of the computation complexity, the degree L's are restricted to 1, 2, 3, 4, and 5.

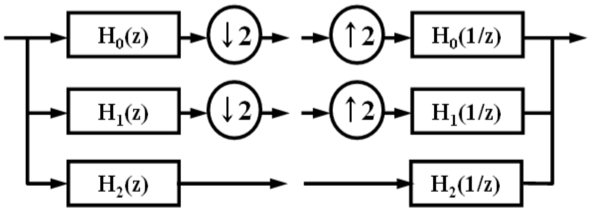

Figure 1. Filterbank of three times oversampling framelet transform

SYMMETRIC TIGHT FRAMELET

Based on Unitary Extension Principle, the anti- or symmetric tight framelets with compact support are constructed. The task for the framelet construction can be reduced to checking two simple conditions due to our definitions of the Laurent symbols. The first condition that the Laurent symbol for the refinable functions should satisfy is

$$A(z) := 2 - H_0(z)H_0(1/z) - H_0(-z)H_0(-1/z) \geq 0 \tag{1}$$

Secondly, if all zeros of A(z) have even multiplicities, we can obtain the Laurent symbol $H_2(z)$ for the second framelet by the spectral factorization. With the Laurent symbol $H_1(z)$ for the first framelet defined by

$$H_1(z) = \begin{cases} z^n H_0(-1/z) & \text{if } n \text{ is odd,} \\ z^{n+1} H_0(-1/z) & \text{if } n \text{ is even,} \end{cases} \tag{2}$$

where n is the degree of H_0, eventually, we get the desired framelet systems.

All the possible anti- or symmetric tight framelets constructed from the quasi-interpolatory subdivision masks of degree up to 5 are presented. We determine the free tension parameter ω's in the subdivision masks of each degree to up 5 whose corresponding refinable functions guarantee the two anti- or symmetric tight framelets. The resulting masks (or filters) in the case of degree 3 are presented in Table 1.

Table 1 In the case of degree L = 3, the lowpass filter h_0 and the highpass filters h_1, h_2 for $\omega = -\frac{1}{64}$ and $\frac{15}{64}$

ω	$-\frac{1}{64}$	$\frac{15}{64}$
h_0	$\frac{\sqrt{2}}{128}\{1,-7,7,63,63,7,-7,1\}$	$\frac{\sqrt{2}}{128}\{-15,9,55,15,15,55,9,-15\}$
h_1	$\frac{\sqrt{2}}{128}\{-1,-7,-7,63,-63,7,7,1\}$	$\frac{\sqrt{2}}{128}\{15,9,-55,15,-15,55,-9,-15\}$
h_2	$\frac{\sqrt{7}}{64}\{1,0,-1,0,-1,0,1,0\}$	$\frac{3\sqrt{15}}{64}\{1,0,-1,0,-1,0,1,0\}$

APPLICATION

Applications to the signal and image denoising and the erasure recovery are presented. To reveal the properties or advantages of the framelet systems, the results of the applications using the Daubechies's orthonomal and biorthogonal wavelet systems are compared with those using the framelet system. For instance, Figure 2 shows the denoised images using each systems. As one can see in Figure 2, our framelet system produces the denoised image with higher quality than

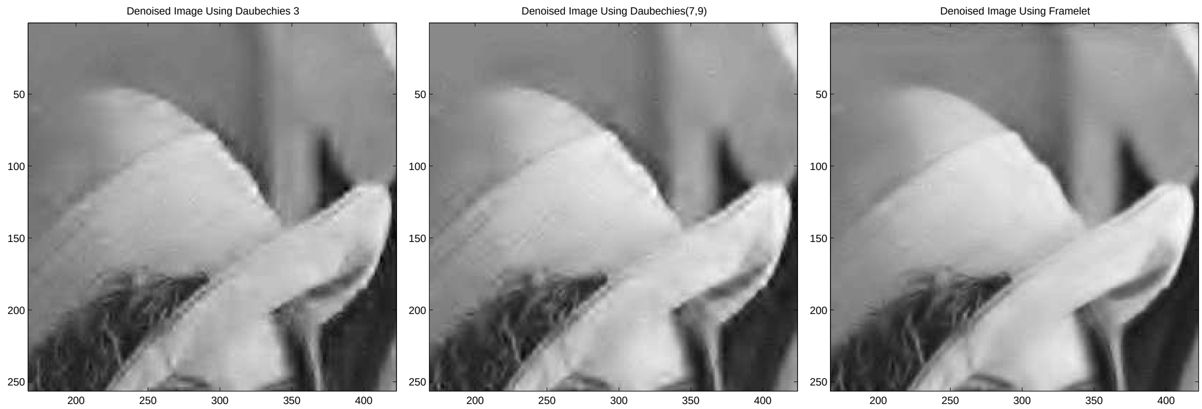

Figure 2. Image denoising results. Left : denoised image using D(3), middle : denoised image using DB(7,9), right : denoised image using F(5,1)

Daubechies's wavelet systems. More specifically, there are fewer distortions of features in the resulting image from the framelet system, while the other images are distorted conspicuously.

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