EXISTENCE AND UNIQUENESS OF VERY SINGULAR SOLUTION FOR THE P-LAPLACIAN EQUATION WITH CONVECTION

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ABSTRACT

We study an existence and uniqueness of very singular self-similar solution for a p-laplacian equation with nonlinear convection term defined on the half line;

\[ u_t = (|u_x|^{p-2}u_x)_x + (u^q)_x \quad \text{in} \quad Q = \mathbb{R}_+ \times \mathbb{R}_+ \]

with homogeneous Neumann boundary condition, where \( p > 2 \) and \( p - 1 < q < 2(p - 1) \). The solution we find is of the form

\[ u(x, t) = t^{-\alpha} f(xt^{-\beta}) := t^{-\alpha} f(r), \quad r = xt^{-\beta}, \]

where \( \alpha := (p - 1)/(pq - 2p + 2) \), \( \beta := (q - p + 1)/(pq - 2p + 2) \) and \( f \) is the nontrivial, nonnegative solution of an nonlinear ordinary differential equation;

\[ (|f'|^{p-2}f')' + \beta rf' + \alpha f + (f^q)' = 0, \quad r > 0 \]

with condition \( f'(0) = 0 \), \( \lim_{r \to \infty} r^{\alpha/\beta} f(r) = 0 \).

REFERENCES


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