

EXISTENCE AND UNIQUENESS OF VERY SINGULAR SOLUTION FOR THE P-LAPLACIAN EQUATION WITH CONVECTION

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ABSTRACT

We study an existence and uniqueness of very singular self-similar solution for a p-laplacian equation with nonlinear convection term defined on the half line;

$$u_t = (|u_x|^{p-2}u_x)_x + (u^q)_x \quad \text{in } \mathbb{Q} = \mathbb{R}^+ \times \mathbb{R}^+$$

with homogeneous *Neumann* boundary condition, where $p > 2$ and $p - 1 < q < 2(p - 1)$. The solution we find is of the form

$$u(x, t) = t^{-\alpha} f(xt^{-\beta}) := t^{-\alpha} f(r), \quad r = xt^{-\beta},$$

where $\alpha := (p - 1)/(pq - 2p + 2)$, $\beta := (q - p + 1)/(pq - 2p + 2)$ and f is the nontrivial, nonnegative solution of an nonlinear ordinary differential equation;

$$(|f'|^{p-2}f')' + \beta r f' + \alpha f + (f^q)' = 0, \quad r > 0$$

with condition $f'(0) = 0$, $\lim_{r \rightarrow \infty} r^{\alpha/\beta} f(r) = 0$.

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