Duality Methods For Total Variation Minimization In Image Processing

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ABSTRACT

Total variation(TV) minimizing models have become one of the most popular and successful methodologies for image restoration since their introduction by Rudin, Osher and Fatemi [1]. The TV-based models can eliminate oscillations(noise) while preserving sharp edges. Recently, TV minimization has also been extended to other image precessing tasks as impaintings, deconvolutions and decompositions et al. In spite of their success, computing numerical solutions of TV models is very challenging. This is mainly due to the non-smoothness, high non-linearity and spatial stiffness of the TV regularization term $\int |\nabla u|$.

In the talk, I'll first give a survey of some existing primal methods. In the original paper[1], Rudin et al. modified TV to smooth $\int \sqrt{|\nabla u|^2 + \beta}$ and solve the associated Euler-Lagrange equation by steepest descent method. The method is very slow due to the time step constraints for parabolic type equations. The fixed-point method developed Vogel et al. is faster but the convergence is still linear. Newton's method is quadratic, but has a very small domain of convergence for the primal TV problem and fails to converge for small β.

In [2], Chan et al. introduced a dual variable $w = \frac{\nabla u}{\sqrt{|\nabla u|^2+\beta}}$ and obtain the primal-dual system. The primal dual system fold out the singularity and nonlinearity onto high dimensions and hence is more suitable to adopt Newton's method. The quadratic convergence for Newton's method is demonstrated in [2] with relatively small β which makes no visual difference. Many other algorithms have also been developed to solve the dual problem ever since. These include local relaxation methods and barrier penalty methods et al. A major advance in this direction was made recently by Chambolle[3], in which he obtained an explicit analytic formula for the Lagrange multipliers for the constraints in the dual problem. It allowed him to develop a gradient descent type method that is global convergent and doesn't need the smooth parameter β. More recently Goldfarb and Yin[4] made an observation that TV minimization model can be transformed into a Second-Order Cone Programming problem(SOCP) and hence can be computed by standard SOCP solvers.

Finally, I will also show the connections between recently developed duality-based methods with our primal-dual Newton method(CGM).