PDE-BASED IMAGE INTERPOLATORS

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ABSTRACT

We present a PDE-based interpolation algorithm to effectively reproduce high resolution imagery. Conventional PDE-based interpolation methods can produce sharp edges without checkerboard effects; however, they are approximators and tend to weaken fine structures. In order to overcome the drawback, a texture enhancement method is incorporated as a post-process of PDE-based interpolation methods. The post-process turns out to make the resulting algorithm an interpolator. It has been numerically verified that the resulting algorithm, called the PDE-based image interpolator (PII), restores sharp edges and enhances fine structures satisfactorily, outperforming the PDE-based skeleton-texture decomposition (STD) approach.

INTRODUCTION

Recently, PDE-based methods have been introduced to constrain continuity of edges and reconstruct appropriate sharp edges through iterations [1–4], beginning from an image interpolated by a conventional interpolation method. These PDE-based methods reproduce sharp edges without checkerboard effects; however, they tend to weaken fine structures in the image, mostly because PDE-based diffusion processes cannot preserve fine structures in a desirable level. Note that the resulting PDE-based algorithm, which is a composite of a conventional interpolation and a PDE-based edge-forming, may not be an interpolator but an approximator.

We will introduce a new texture enhancement method, which can be easily incorporated as a post-process of PDE-based interpolation methods [1,2]. Here our idea is simple: Add a post-process in order to make the overall operation an interpolator.

PRELIMINARIES

This section begins with a brief review on the PDE-based edge-forming procedure suggested by the authors [1,2], followed by the PDE-based skeleton-texture decomposition (STD) approach [5].

The PDE-Based Edge-Forming Method

A color image $I = (r, g, b)$ can be decomposed into brightness and chromaticity:

$$\eta = \sqrt{r^2 + g^2 + b^2}, \quad \varphi = \tan^{-1}\left(\frac{b}{\sqrt{r^2 + g^2}}\right), \quad \xi = \tan^{-1}\left(\frac{g}{r}\right),$$

where η is the brightness, and φ and ξ are the chromaticity components. It has been verified that in image restoration, the use of the chromaticity-brightness (CB) decomposition results in better restored images than conventional approaches such as the channel-by-channel model and the HSV system.

To get an edge-forming model, we use the following [1]:

$$\begin{aligned}
\frac{\partial \eta}{\partial t} - |\nabla \eta|^q \, \nabla \cdot \left(\frac{\nabla \eta}{|\nabla \eta|^q}\right) &= \beta_\eta(\eta^0 - \eta), \\
\frac{\partial \varphi}{\partial t} - \mathcal{R}(\varphi, \xi)^q \, \nabla \cdot \left(\frac{1}{\mathcal{R}(\varphi, \xi)^q} \nabla \varphi\right) &= \sin\varphi \, \cos\xi \, |\nabla \xi|^2 \\
&\quad + \beta_\varphi(\varphi^0 - \varphi), \\
\frac{\partial \xi}{\partial t} - \mathcal{R}(\varphi, \xi)^q \, \nabla \cdot \left(\frac{\cos^2 \varphi}{\mathcal{R}(\varphi, \xi)^q} \nabla \xi\right) &= \beta_\xi(\xi^0 - \xi),
\end{aligned} \tag{1}$$

where $(\eta^0, \varphi^0, \xi^0)$ denotes an initialization of (η, φ, ξ) and

$$\beta_\phi \; = \; \alpha \, |\Delta \, \phi^0|^\delta, \quad \phi = \eta, \, \varphi, \, \xi.$$

Here α and δ are positive parameters to be determined. The equations in the model (1) can be solved effectively by employing a linearized Crank-Nicolson scheme along with the alternating directional implicit (ADI) procedure. See [1,2] for details.

It has been numerically verified that the model (1) incorporating the suitable numerical schemes can form reliable edges satisfactorily and efficiently. However, it tends to weaken fine structures. For gray-scale images, the first equation in (1) can be applied, without any modifications, for an effective edge-forming.

The PDE-Based Skeleton-Texture Decomposition Approach

The interpolation method to be presented in this subsection focuses on the interpolation of gray-scale images. For color images, the method can be applied in the channel-by-channel fashion to each of the RGB system, the HSV system, or the CB transformation.

The interpolation method suggested by Saito *et al.* [5] begins with the multiplicative skeleton-texture decomposition (STD)

$$I = U \cdot V + D, \tag{2}$$

where U is the skeleton image, V denotes the texture generator, and D is the construction error. For the decomposition, the authors applied the additive STD method (in the logarithm domain) suggested by Vese and Osher [6]. Once the decomposition is performed, the three components are interpolated separately. The interpolated image of I, I', is obtained as

$$I' = U' \cdot V' + D', \tag{3}$$

where U' is the interpolation of U carried out by applying the TV-based deblurring-oversampling approach [4], D' denotes a statistically resampled interpolation of D, and V' is the bilinear interpolation of a variation of V, $\hat{V}$:

$$\hat{V} = \exp(\kappa(v)), \quad \kappa(v) = v\,[1 + (\alpha_M - 1)\rho(v)], \quad \alpha_M \geq 1,$$
$$\rho(v) = \begin{cases} (1 - v^2/c^2)^2, \text{ if } |v| < c, \\ 0, \qquad\qquad\quad \text{ if } |v| \geq c. \end{cases} \tag{4}$$

The algorithm (4) has been designed to enhance texture components, but not to introduce overshooting on texture-packed regions. Thus it enhances texture components more strongly on relatively slow transitions. However, since (4) is not an interpolator but an approximator, the resultant image can be easily blurry, and the choice of c and α_M is problematic.

THE PDE-BASED IMAGE INTERPOLATOR

For a new texture-enhancing interpolation algorithm, we will begin with the PDE-based edge-forming method presented in the previous section and try to make the overall algorithm an *interpolator* by simply incorporating the bilinear interpolation of the texture components missed during the edge-forming.

Let I^* be the solution of the PDE-based edge-forming method presented in 1. Then it can be written as

$$I^* = \mathcal{P} I_L^H I, \tag{5}$$

where I is a given image of low resolution, I_L^H denotes a conventional interpolation method from the low resolution to the high resolution (zoom-in), and $\mathcal{P}$ is the operator of the PDE-based edge-forming. Let I_H^L be a zoom-out operator, the dual of I_L^H.

We suggest the following algorithm for the texture enhancement:

$$\begin{aligned} &\text{(a) } u \;= I_H^L\, I^*, \\ &\text{(b) } v \;= I \ominus u, \\ &\text{(c) } I' = I^* \oplus I_L^H v, \end{aligned} \tag{6}$$

where I' is the final result, $\oplus$ denotes either addition (+) or multiplication $(\cdot)$, and $\ominus$ is subtraction $(-)$ or division $(\div)$. The functions u and v can be viewed respectively as the skeleton image and the texture generator of I. In this article, the resulting algorithm will be called the *PDE-based image interpolator* (PII). When an integer magnification is considered, the final result I' will have the same values as I at the grids of the low resolution image. Thus PII is an interpolator.

NUMERICAL EXPERIMENTS

In Figure 1, we present zoomed images of (Tiffany's) Eye processed by the two algorithms. As one can see from the figure, the new algorithm has reproduced clearer texture components and better contrasts; see particularly around the eyebrow and the eyelash. This example shows

(a) (b)

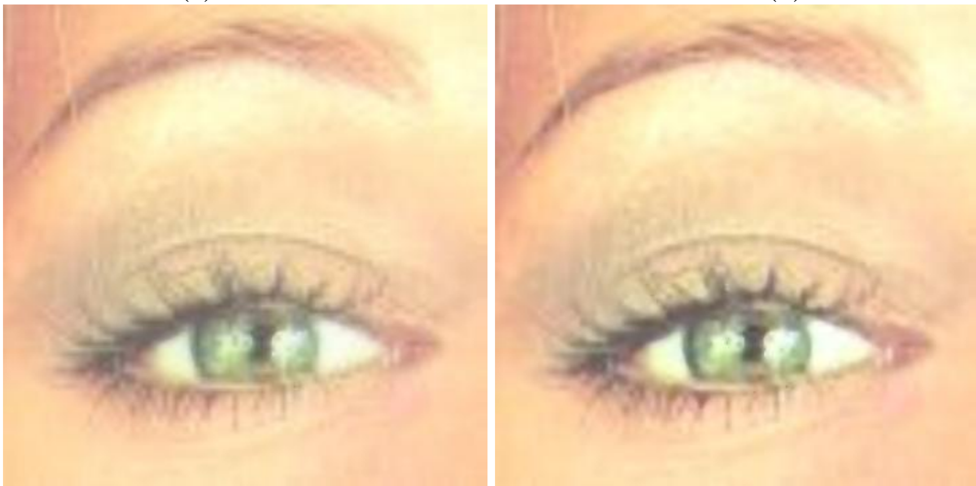

Figure 1. Eye: (4×4)-magnified images by (a) the STD approach with $\alpha_M = 1$ and (b) PII.

again that the new texture-enhancing interpolation algorithm (an interpolator) can reproduce zoomed images more satisfactorily than the STD approach, an approximator.

It should be noticed that the STD approach is more expensive than PII, due to the additional operation of skeleton-texture decomposition which solves the associated Euler-Lagrange equations. See [6] for details.

CONCLUSIONS

We have introduced a simple texture-enhancement technique to be employed as a post-process of conventional PDE-based methods. The texture enhancement has been carried out for the resultant image not to modify the image values at the grids of the low resolution image. From numerical experiments, we have reached the conclusion that a PDE-based edge-forming method can be effective in the interpolation and satisfactory in the texture enhancement, when it is simply an interpolator.

REFERENCES

1. Y. Cha and S. Kim, "Edge-forming methods for color image zooming," *IEEE Trans. Image Process.*, vol. 15, no. 8, pp. 2315–2323, 2006.
2. ——, "Edge-forming methods for image zooming," *J. Math. Imaging and Vis.*, vol. 25, no. 3, pp. pp. 353–364, 2006.
3. F. Guichard and F. Malgouyres, "Total variation based interpolation," in *Proceedings of the Ninth European Signal Processing Conference*, Patran, Greece, 1998, pp. 1741–1744.
4. F. Malgouyres and F. Guichard, "Edge direction preserving image zooming: A mathematical and numerical analysis," *SIAM J. Numer. Anal.*, vol. 39, pp. 1–37, 2001.
5. T. Saito, Y. Ishii, Y. Nakagawa, and T. Komatsu, "Adaptable image interpolation with skeleton-texture separation," in *Proceedings of the 13th IEEE International Conference on Image Processing*, 2006, pp. 681–684.
6. L. Vese and S. Osher, "Modeling textures with total variation minimization and oscillating patterns in image processing," *J. Sci. Comput.*, vol. 19, pp. 553–572, 2003.