B-SPACES AND THEIR CHARACTERIZATION VIA ANISOTROPIC FRANKLIN BASES

G. KYRIAZIS [1] , K. PARK [2] and P. PETRUSHEV [3]

1) *Department of Mathematics and Statistics, University of Cyprus, Nicosia, CYPRUS*

2) *Department of Mathematics, Ewha W. University, Seoul, KOREA*

3) *Department of Mathematics, University of S. Carolina, Columbia, SC, USA*

Corresponding Author : Kyungwon PARK, parkusc@gmail.com

ABSTRACT

B-spaces (generalized Besov spaces) generated by multilevel nested triangulations of compact polygonal domains in $\mathbb{R}^2$ are explored. Mild conditions are imposed on the triangulations which prevent them from deterioration and at the same time allow for a lot of flexibility and, in particular, arbitrarily sharp angles. It is shown that the B-spaces can be characterized by the corresponding anisotropic Franklin bases. This result is applied to nonlinear n-term approximation from anisotropic Franklin bases.

KEYWORDS

Nonlinear approximation, n-term approximation, multilevel nested triangulations, Franklin Bases, B-spaces.

INTRODUCTION

We consider general B-spaces generated by sequences of multilevel nested triangulations of compact polygonal domains in $\mathbb{R}^2$. For a given polygonal domain E in $\mathbb{R}^2$ we consider a sequence of nested triangulations $\mathcal{T}_0, \mathcal{T}_1, \dots$ of general nature. Mild conditions are imposed on the triangulations which prevent them from deterioration. At the same time these conditions allow for a great deal of flexibility and, in particular, arbitrarily sharp angles. Generalized Besov spaces B^α_{pq} (called B-spaces) are naturally associated with every such sequence of triangulations and provide a specific nonstandard means of measuring the smoothness of the functions.

In this talk, we show that the general B-spaces can be characterized via Franklin bases obtained by applying the Gram-Schmidt orthogonalization process to the corresponding Courant elements. Further, we show how the B-spaces can be used to characterize the approximation spaces of nonlinear n-term approximation from anisotropic Franklin systems in L_p ($1 < p < \infty$).

REFERENCES

1. R. DEVORE, Nonlinear approximation, *Acta Numerica* **7** (1998), 51–150.

2. C. FEFFERMAN AND E. STEIN, Some maximal inequalities, *Amer. J. Math.* **93** (1971), 107–115.

3. A. JONSSON AND A. KAMONT, Piecewise linear bases and Besov spaces on fractal sets, *Anal. Math.* **27** (2001), 77–117.

4. B. Karaivanov and P. Petrushev, Nonlinear piecewise polynomial approximation beyond Besov spaces, *Appl. Comput. Harmon. Anal.* **15** (2003), 177–223.

5. B. KARAIVANOV, P. PETRUSHEV, AND R. C. SHARPLEY, Algorithms for nonlinear piecewise polynomial approximation, *Trans. Amer. Math. Soc.* **355** (2003), 2585–2631.

6. P. PETRUSHEV AND V. POPOV, "Rational Approximation of Real Functions", Cambridge University Press, 1987.