TOPOLOGY DETERMINATION OF CRITICAL CASES IN SURFACE-SURFACE INTERSECTION

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ABSTRACT

Determining the topology of intersection curves is one of the important issue of surface-surface intersection problem used in Computer Aided Geometric Design. To compute the intersection curves, we first need to determine the topology of the curves. Thomas A. Grandine[1] presented an algorithm to determine topology using partial derivatives of surface intersection equations. When the two surfaces meet tangentially, some differentiated values of surface parameters are not exactly determined. This case is called a critical case. In this case, we find exact solution of uncertain differentiated values using the tangent vector of the intersection curve.

INTRODUCTION

Let S_1 and S_2 be two algebraic surfaces in $\mathbb{R}^3$ definded by the following parametric representations

$$S_1 = \{\mathbf{f}(u,v) = (f^x(u,v), f^y(u,v), f^z(u,v)) : 0 \le u, v \le 1\},$$
$$S_2 = \{\mathbf{g}(s,t) = (g^x(s,t), g^y(s,t), g^z(s,t)) : 0 \le s, t \le 1\}.$$

If two surfaces come in contact on the intersection line $l = S_1 \cap S_2$ (two surfaces have the same tangent plane on l), we can not decide the correct vector $(\frac{du}{ds}, \frac{dv}{ds}, \frac{dt}{ds})$ at the point $s = 0$ by the equation

$$\frac{d}{ds}(\mathbf{f}(u,v) - \mathbf{g}(s,t)) = 0.$$

This equation can be written of the form

$$\begin{pmatrix} \mathbf{f}_u & \mathbf{f}_v & -\mathbf{g}_t \end{pmatrix} \begin{pmatrix} \frac{du}{ds} \\ \frac{dv}{ds} \\ \frac{dt}{ds} \end{pmatrix} = \begin{pmatrix} \mathbf{g}_s \end{pmatrix}.$$

When two surfaces meet tangentially, the determinant of the matrix $\begin{pmatrix} \mathbf{f}_u & \mathbf{f}_v & -\mathbf{g}_t \end{pmatrix}$ is zero. Therefore we can not decide the vector $(\frac{du}{ds}, \frac{dv}{ds}, \frac{dt}{ds})$. We present a method of finding the correct vector $(\frac{du}{ds}, \frac{dv}{ds}, \frac{dt}{ds})$. Using this vector, we can decide the characteristic of the point $s = 0$.

FINDING DIFFERENTIATED VALUES

Since two tangent planes coincide on the point $s=0$, we have an invertible matrix A

$$A=\begin{pmatrix}\alpha & \gamma\\ \beta & \delta\end{pmatrix}$$

such that

$$\begin{pmatrix}\mathbf{f}_u & \mathbf{f}_v\end{pmatrix}\begin{pmatrix}\alpha & \gamma\\ \beta & \delta\end{pmatrix}=\begin{pmatrix}\mathbf{g}_s & \mathbf{g}_t\end{pmatrix}$$

and, consequently, we have

$$\begin{pmatrix}\frac{du}{ds}\\ \frac{dv}{ds}\end{pmatrix}=\begin{pmatrix}\alpha & \gamma\\ \beta & \delta\end{pmatrix}\begin{pmatrix}1\\ \frac{dt}{ds}\end{pmatrix}=\begin{pmatrix}\alpha & \gamma\\ \beta & \delta\end{pmatrix}\begin{pmatrix}\frac{ds}{ds}\\ \frac{dt}{ds}\end{pmatrix}.$$

Thereupon,

$$\frac{du}{ds}=\alpha+\gamma\frac{dt}{ds}$$
$$\frac{dv}{ds}=\beta+\delta\frac{dt}{ds}$$

and, with this, we have the parameterized form of u and v with s:

$$u=u_0+\left(\alpha+\gamma\frac{dt}{ds}\right)s+O\left(s^2\right),$$
$$v=v_0+\left(\beta+\delta\frac{dt}{ds}\right)s+O\left(s^2\right).$$

With this we find the tangent vector of the intersection curve at $s=0$ and, consequently, we can find the value $\frac{dt}{ds}$ at $s=0$. In fact, on the parametric surface curve

$$\mathbf{g}\left(s,t(s)\right)=\left(g^x\left(s,t(s)\right),g^y\left(s,t(s)\right),g^z\left(s,t(s)\right)\right):0\leq s\leq 1,$$

if we know the tangent vector $\mathbf{t}=(t_x,t_y,t_z)$ at $s=0$, we can find $\frac{dt}{ds}$ by the equality

$$\frac{d}{ds}\mathbf{g}\left(s,t(s)\right)=\mathbf{t}=(t_x,t_y,t_z).$$

Using this value we can decide starting or ending of the point which is on the intersection curve at $s=0$.

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