A THREE-LEVEL BDDC ALGORITHM FOR MORTAR DISCRETIZATIONS

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ABSTRACT

In this talk, three-level BDDC algorithms will be presented for the solutions of large sparse linear algebraic systems arising from the mortar discretization of elliptic boundary value problems. The mortar discretization is considered on geometrically non-conforming subdomain partitions. In the algorithms, the large coarse problems from two-level BDDC algorithms are solved approximately while a good rate of convergence is maintained. This is an extension of previous work for the three-level BDDC algorithms with standard finite element discretization by Tu [12,13]. Estimates of the condition numbers are provided for the three-level BDDC methods and numerical experiments are also discussed.

INTRODUCTION

Mortar methods were introduced by Bernardi, Maday, and Patera [14] to couple different approximations in different subdomains so as to obtain a good global approximate solution. They are useful for modelling multi-physics, adaptivity, and mesh generation for three dimensional complex structures. The coupling is done by enforcing certain constraints on solutions across the subdomain interface using Lagrange multipliers. We call these constraints the mortar matching conditions. Both dual and primal forms can be obtained from the mortar discretization. Since the dual form is similar to the linear systems solved by FETI-DP algorithms, FETI–DP algorithms have been developed for solving the dual form of mortar discretizations [5,3]. Later BDDC algorithms, that are closely related to the FETI–DP algorithms, were also developed for solving the primal form of the mortar discretization [4]

BDDC (Balancing Domain Decomposition by Constraints) methods were introduced and analyzed in [1,10,11] for standard finite element discretizations. These iterative methods are new versions of the balancing Neumann-Neumann algorithms with a new coarse problem given in terms of a set of primal constraints. Two–level BDDC methods have been extended to mortar finite element discretization in [4]. The edge/face primal constraints are considered, which lead to much larger coarse problems than standard discretizations. This is due to the geometrically non-conforming subdomain partitions and the fact that only face primal constrains are possible for three dimensions. In the two-level BDDC algorithms, the coarse problems are generated and factored by direct solvers at the beginning of the computation. The coarse components can be the bottleneck of the algorithms if the number of the subdomains is large.

Recently, there are several papers about inexact solvers for BDDC algorithms with standard finite element discretization. In [12,13], two three-level BDDC algorithms are introduced which solve the coarse problems inexactly by introducing an additional level. Inexact local solvers based on multigrid methods were introduced by Li and Widlund in [9]. In [15], several inexact solvers for both the coarse and the local components are considered. An inexact FETI-DP algorithm is also introduced in [7].

In this paper, we extend the algorithms in [12,13] to mortar finite element discretization. We solve the coarse problem approximately, by introducing an additional level and using the BDDC idea recursively, and show that a good rate of convergence still can be maintained. We also provide estimates of the condition number bounds of the system with the new preconditioners.Numerical results will be presented for both two and three–dimensional elliptic problems.

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