A MUSIC-TYPE ALGORITHM FOR DETECTING INTERNAL CORROSION FROM ELECTROSTATIC BOUNDARY MEASUREMENTS

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ABSTRACT

We establish an asymptotic representation formula for the steady state currents caused by internal corrosive parts of small Hausdorff measures. Based on this formula we design a non-iterative method of MUSIC (multiple signal classification) type for localizing the corrosive parts from voltage-to-current observations.

INTRODUCTION

In the field of nondestructive evaluation, new and improved techniques are constantly being sought to facilitate the detection of hidden corrosion in pipelines. Hidden corrosion and defects can cause serious problems and is responsible for millions of dollars annually in cost of replacement infrastructure and lost production, and is a dangerous threat to safety and the environment. It is of great importance to detect and to quantify most unseen potential hazards before they become problems.

Corrosion occurs in many different forms and several different models can be encountered in the literature (see, for example, Kaup and Santosa [15], Kaup et al. [16], Vogelius and Xu [20], Inglese [12], Luong and Santosa [13], Banks et al. [3] and references therein).

In this paper, we adopt the potential model: Laplace's equation holds in the pipeline and the effect of corrosion is described by means of Robin boundary conditions. The study of a model like this is motivated by a number of favorable indications. A first indication is based on the observation that corrosion tends to roughen a surface: in fact, this effect can be modelled by the introduction of a thin coating characterized by rapid oscillations. In the limit where the thickness of the coating goes to zero and the rapidity of the oscillations diverges, the arising of Robin boundary conditions has been observed by Buttazo and Kohn [5]. On the other hand, the study of electrochemical corrosion processes can be based on Faraday's law which says that the mass loss which is a measure of corrosion is proportional to the normal current flux. In Vogelius and Xu [20] a potential model of this kind of process is proposed. If we linearize with respect to the transfer coefficient the nonlinear boundary conditions in [20], we get Robin boundary conditions.

The inverse problem of corrosion detection consists of the determination of the corrosion damage of an inaccessible part of the surface of a specimen when the available data are a finite number of voltage-to-current pairs on the accessible part. Difficulties of this inverse problem result from its inherent ill-posedness and nonlinearity. Most of the techniques for detecting the corrosion are based on iterative algorithms: least-square algorithms and Newton-type iteration schemes. In these methods, one needs tremendous computational costs and time to get a close image to the true solution, since these iterative algorithms may not converge to an approximate solution.

The purpose of this work is to design a direct (non-iterative) technique for detecting corrosion in pipelines from voltage-to-current observations. Our new algorithm is of MUSIC-type (multiple signal classification) and is based on an accurate asymptotic representation formula for the steady state current perturbations.

REFERENCES

1. Ammari, H. and Kang, H., "High-order terms in the asymptotic expansions of the steady-state voltage potentials in the presence of conductivity inhomogeneities of small diameter", *SIAM J. Math. Anal.*, Vol. 34, 2003, pp. 1152-1166.
2. Ammari, H., Moskow S., and Vogelius M.S., "Boundary integral formulas for the reconstruction of electromagnetic imperfections of small diameter", *ESAIM: Cont. Opt. Calc. Var.*, Vol. 9 ,2003, pp. 49-66.
3. Banks H.T., Joyner M.L., Wincheski B., and WinfreeW.P., "Real time computational algorithms for eddy-current-based damage detection, Inverse Problems", Vol. 18, 2002, pp. 795-823.
4. Brühl M., Hanke M. and Vogelius M.S., "A direct impedance tomography algorithm for locating small inhomogeneities", *Numer. Math.*, Vol. 93, 2003, pp. 635-654.
5. Buttazzo G. and Kohn R.V., "Reinforcement by a thin layer with oscillating thickness", *Appl. math. Opt.*, Vol. 16, 1988, pp. 247-261.
6. Cedio-Fengya D.J., Moskow S. and Vogelius M.S., "Identification of conductivity imperfections of small diameter by boundary measurements. Continuous dependence and computational reconstruction", *Inverse Problems*, Vol. 14, 1998, pp. 553-595.
7. Cheney M., "The linear sampling method and the MUSIC algorithm", *Inverse Problems*, Vol. 17, 2001, pp. 591-595.
8. Coifman R.R., McIntosh A., and Meyer Y., "L'intégrale de Cauchy définit un opérateur borné sur L^2 pour les courbes lipschitziennes", *Ann. of Math. (2)*, Vol. 116, 1982, pp. 361-387.
9. Colton D. and Kirsch A., "A simple method for solving inverse scattering problems in the resonance region", *Inverse Problems*, Vol. 12, 1996, pp. 383-393.
10. Devaney A.J., "Time reversal imaging of obscured targets from multistatic data", *IEEE Trans. Antennas Propagat.*, Vol. 523, 2005, pp. 1600-1610.
11. Folland G.B., *Introduction to Partial Differential Equations*, Princeton University Press, Princeton, NJ, 1976.

12. Inglese G., “An inverse problem in corrosion detection”, *Inverse Problems*, Vol. 13, 1997, pp. 977-994.

13. Luong B.and Santosa F., “Quantitative imaging of corrosion inplates by eddy current methods”, *SIAM J. Appl. Math.*, Vol. 58, 1998, pp. 1509-1531.

14. Kang H. and Seo J.K., “Layer potential technique for the inverse conductivity problem”, *Inverse Problems*, Vol. 12, 1996, pp. 267-278.

15. Kaup P. and Santosa F, “Nondestructive evaluation of corrosion damage using electrostatic measurements”, *J. Nondestructive Eval*. Vol. 14, 1995, pp. 127-136.

16. Kaup P., Santosa F., and Vogelius M., “A method for imaging corrosion damage in thin plates from electrostatic data”, *Inverse Problems*, Vol. 12, 1996, pp. 279-293.

17. Kirsch A, “The MUSIC algorithm and the factorisation method in inverse scattering theory for inhomogeneous media”, *Inverse Problems*, Vol. 18, 2002, pp. 1025-1040.

18. Stein E.M., *Singular Integrals and Differentiability Properties of Functions”*, Princeton University Press, Princeton, NJ, 1970.

19. Therrien C.W., *Discrete Random Signals and Statistical Signal Processing*, Prentice-Hall, Englewood Cliffs, NJ, 1992.

20. Vogelius M. and Xu J., “A nonlinear elliptic boundary value problem related to corrosion modelling”, *Quart. Appl. Math.*, Vol. 56, 1998, pp. 479-505.

21. Yang X., Choulli M., and Cheng J., “An iterative BEM for the inverse problem of detecting corrosion in a pipe”, *Numer. Math. J. Chinese Univ.*, Vol. 14, 2005, pp. 252-266.