THE MATHEMATICAL MODELING AND NUMERICAL SIMULATIONS FOR THE MOTION OF SOAP BUBBLES

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ABSTRACT

In this paper, we develop the appropriate equations for bubble motions depending on curvature using the level set methodology. Our method consists of an appropriate finite difference scheme for solving our model equations and a level set approach for capturing the complicated motion between the bubbles. Results indicate that our models and the level set methods can handle complicated interfacial motions and topology changes, and that they can numerically simulate many of the physical features of bubble motions.

INTRODUCTION

In this paper, we will derive the proper model equations and numerical algorithms for soap bubbles. It is generally accepted that many researchers in the field of CFD have difficulties in dealing with singularities of fluid interfaces . To resolve these difficulties, we will use a level set approach to compute the motion of soap bubbles. We intend to capture the interface using a level set approach instead of explicitly tracking it. Front tracking methods usually require that we add or subtract points dynamically. In [2], an interesting front tracking method which does not explicitly reposition the points of interface was devised. The investigators cited good results, but it seems hard to implement in three space dimensions. Moreover, topological changes cause difficulties, as with all tracking methods. In [4], a level set formulation for moving interfaces with curvature dependent velocity was introduced. The level set function is typically a smooth function, denoted as ϕ. The level set formulation eliminates the problem of repositioning the points to a moving grid and is capable of capturing geometric properties of highly complicated boundaries including topological changes without explicitly tracking the interfaces. An application of the level set formulation was used in ([8], [6]) for incompressible fluid flows. They found that it was best, at least close to the front, to keep ϕ as the signed distance from the front to prevent the development of steep or flat gradients in ϕ. This can be done by solving a simple initial value problem for ϕ which leaves the front location unchanged. We will use the level set approach to solve the problem for motion of soap bubbles in 2D and 3D. This includes area preserving motion by velocity or by acceleration.

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