COMPACTLY SUPPORTED SYMMETRIC TIGHT WAVELET FRAMES CONSTRUCTED FROM QUASI-INTERPOLATORY SUBDIVISION MASKS

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ABSTRACT

Multiresolution analysis (MRA) based construction of the tight wavelet frames using fewer generators with symmetricity and compact support is one of the most important matters in theory and application where the redundant function representation is necessary. Especially, those wavelet frames associated with the interpolatory MRA are of interest. However, there is only one such MRA, generated by the piecewise linear B-spline refinable function. As an extension, this paper presents a class of anti- or symmetric tight wavelet frames with compact support based on the quasi-interpolatory MRA. The wavelet frames are constructed from the quasi-interpolatory subdivision masks whose refinable functions reproduce polynomials up to certain degree. Essentially two wavelet frame generators with the three times oversampling framelet transform are employed to reduce the shift variance of the discrete framelet transform and to increase the redundancy of the frequency sampling. The framelet transform can filter the exact intermediate frequency band between low and high frequencies, which guarantees the ternary frequency scales. Applications to signal and image denoising and erasure recovery revealing these properties of the wavelet frames are provided.

INTRODUCTION

The existence of two tight wavelet frame generators (or mother framelets) with symmetricity and compact support associated with the interpolatory MRA was revealed by Petukhov in [11]. Petukhov proved that an interpolatory symbol H_0 admits anti- or symmetric solutions to UEP if and only if $H_0(z) = (z^{1-2N} + 2 + z^{2N-1})/4\sqrt{2}$. These solutions generate a refinable function only when $N = 1$ since the cascade algorithm does not converge in all the other cases [9]. In this case, the refinable function which generates the interpolatory MRA is the piecewise linear B-spline with C^0 smoothness. Motivated by this result, we consider the quasi-interpolatory subdivision masks with a tension parameter in [3] as the lowpass filters. We study the quasi-interpolatory subdivision masks for the construction of anti- or symmetric compactly supported tight wavelet frames with essentially two frame generators using a three times oversampling filter bank as Figure 1. For three times oversampling framelet transform, we need three frame generators one of which is half integer shift version of another generator, which explains the meaning of ‘essentially two frame generators’. Because of the computation complexity, the degree L’s are restricted to 1, 2, 3, 4, and 5.

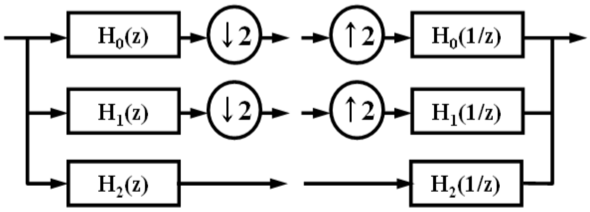

Figure 1. Filterbank of three times oversampling framelet transform

SYMMETRIC TIGHT FRAMELET

Based on Unitary Extension Principle, the anti- or symmetric tight framelets with compact support are constructed. The task for the framelet construction can be reduced to checking two simple conditions due to our definitions of the Laurent symbols. The first condition that the Laurent symbol for the refinable functions should satisfy is

$$A(z) := 2 - H_0(z)H_0(1/z) - H_0(-z)H_0(-1/z) \geq 0 \tag{1}$$

Secondly, if all zeros of A(z) have even multiplicities, we can obtain the Laurent symbol $H_2(z)$ for the second framelet by the spectral factorization. With the Laurent symbol $H_1(z)$ for the first framelet defined by

$$H_1(z) = \begin{cases} z^n H_0(-1/z) & \text{if } n \text{ is odd,} \\ z^{n+1} H_0(-1/z) & \text{if } n \text{ is even,} \end{cases} \tag{2}$$

where n is the degree of H_0, eventually, we get the desired framelet systems.

All the possible anti- or symmetric tight framelets constructed from the quasi-interpolatory subdivision masks of degree up to 5 are presented. We determine the free tension parameter ω's in the subdivision masks of each degree to up 5 whose corresponding refinable functions guarantee the two anti- or symmetric tight framelets. The resulting masks (or filters) in the case of degree 3 are presented in Table 1.

Table 1 In the case of degree L = 3, the lowpass filter h_0 and the highpass filters h_1, h_2 for $\omega = -\frac{1}{64}$ and $\frac{15}{64}$

ω	$-\frac{1}{64}$	$\frac{15}{64}$
h_0	$\frac{\sqrt{2}}{128}\{1, -7, 7, 63, 63, 7, -7, 1\}$	$\frac{\sqrt{2}}{128}\{-15, 9, 55, 15, 15, 55, 9, -15\}$
h_1	$\frac{\sqrt{2}}{128}\{-1, -7, -7, 63, -63, 7, 7, 1\}$	$\frac{\sqrt{2}}{128}\{15, 9, -55, 15, -15, 55, -9, -15\}$
h_2	$\frac{\sqrt{7}}{64}\{1, 0, -1, 0, -1, 0, 1, 0\}$	$\frac{3\sqrt{15}}{64}\{1, 0, -1, 0, -1, 0, 1, 0\}$

APPLICATION

Applications to the signal and image denoising and the erasure recovery are presented. To reveal the properties or advantages of the framelet systems, the results of the applications using the Daubechies's orthonomal and biorthogonal wavelet systems are compared with those using the framelet system. For instance, Figure 2 shows the denoised images using each systems. As one can see in Figure 2, our framelet system produces the denoised image with higher quality

Figure 2. Image denoising results. Left : denoised image using D(3), middle : denoised image using DB(7,9), right : denoised image using F(5,1)

than Daubechies's wavelet systems. More specifically, there are fewer distortions of features in the resulting image from the framelet system, while the model's hat, nose, and lip in the other images are distorted conspicuously.

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