DEVELOPMENT OF REPETTITIVE RESPONSE SURFACE ENHANCEMENT TECHNIQUE FOR THE MULTIDISCIPLINARY OPTIMIZATION

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ABSTRACT

In this study, repetitive response surface enhancement technique (RRSET) is proposed as a new system approximation method for the efficient multidisciplinary design and optimization (MDO). In order to represent the highly nonlinear behavior of the response with second order polynomials, RRSET introduces a design space transformation using stretching functions and repetitive response surface improvement. The tentative optimal point is repetitively included to the set of experimental points to better approximate the response surface of the system especially near the optimal point, hence a response surface with significantly improved accuracy can be generated with very small experimental points and system iterations. As a system optimizer, the simulated annealing, a global optimization algorithm is utilized. The proposed technique is applied to the several numerical examples, and demonstrates the validity and efficiency of the method. With its improved approximation accuracy, the RRSET can contribute to resolve large and complex system design problems under MDO environment.

INTRODUCTION

Recently the computer codes and the analysis methods, required for the engineering design are getting more complex. In most cases, the responses obtained from analyses of either a single discipline or multi-disciplines through the system approach, have numerical noises, irregularities and discontinuities, which make it difficult to get the gradient information and cause to increase the computational loads[1].

Therefore, studies on the approximation techniques to resolve these issues are very important approach in the system design using MDO[2].

The response surface method (RSM)[3] is a statistical method, utilizing the Design of Experiment (DOE) theory. The RSM which approximates the design space as first or second order polynomials is relatively simple and many approaches to increase the accuracy of the approximation have been developed, hence the RSM is frequently used in many engineering design and optimization applications[1,4,5,6,7] But in spite of the many strong points of the approximation using the response surfaces, they have some limitations[6].

In this study, in order to overcome the limitations of the approximation using the response surface, hence to design and optimize given engineering problems efficiently, the Repetitive Response Surface Enhancement Technique (RRSET) will be proposed. The RRSET will be applied to numerical examples of which the exact solutions are known, and two member frame design and optimization problem, then the validity and the superiority of the method will be demonstrated.

REPETATIVE RESPONSE SURFACE ENHANCEMENT TECHNIQUE (RRSET)

Usually a quadratic function is utilized to build a regression model. As the number of design variables increases, the number of coefficients of the regression model, hence the number of required experimental points increases exponentially. Therefore, to increase the order of the regression function above order 2 is not practically useable. Different approach that emphasizes the highly nonlinear design area (or the region of high gradient) is proposed by the authors: The design space transformation technique utilizing the design variable stretching. Proper selection of the stretching location (***D***) and the stretching amount (β) is important for the optimal construction of the response surface.

$$x_{ci} = D\left[1 + \frac{\sinh\{\beta(x_{ni} - A)\}}{\sinh(\beta A)}\right] \quad Where\ , \qquad A_i = \frac{1}{2\beta_i}\ln\left[\frac{1 + \left(e^{\beta_i} - 1\right)\left(D_i/H\right)}{1 + \left(e^{-\beta_i} - 1\right)\left(D_i/H\right)}\right] \qquad (1)$$

Where, β is the stretching parameter and D is the stretching position. H_i is range of the i-th design variable, and subscript i denotes the design variable index.

To generalize the design space transformation using stretching, RSM (Response Surface Model) Optimizer, a sub-optimization for the stretching parameter is included in the overall optimization process. The stretching level β_i and the stretching position D_i of i-th design variable are new design variables and adjusted R-squared(R^2_{adj}), a criterion of the confidence level, is selected as the objective function to be maximized.

$$Maximize \quad R^2_{adj} = f(\boldsymbol{\beta}, \boldsymbol{D}) \qquad where, \quad \boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_k \end{pmatrix}, \quad \boldsymbol{D} = \begin{pmatrix} D_1 \\ \vdots \\ D_k \end{pmatrix} \qquad (2)$$

SYSTEM OPTIMIZATION PROCEDURE

To minimize the number of analysis runs while maintaining the reliable regression model, the initial response surface is built with relatively small number of experimental points. Then, the tentative optimum point obtained using the regression model, is included to the next set of experimental points to build a new regression model. Tentative optimum points (additional experiment points at each design cycle) are kept adding until design convergence. Usually two or three design iterations are enough to obtain the converged optimum solution. With only one additional analysis run at each design iteration, quality regression model can be constructed. Overall system optimization procedure including the sub-optimization is shown at fig. 1. For the RSM optimizer, a SA (Simulated Annealing) algorithm[8] is implemented to obtain global optimum.

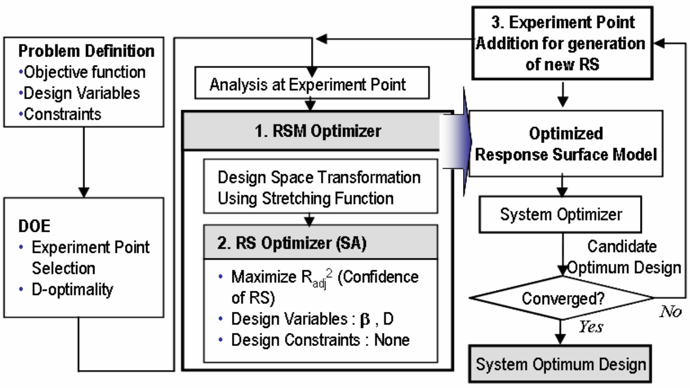


Figure 1. System Optimization Flow Chart

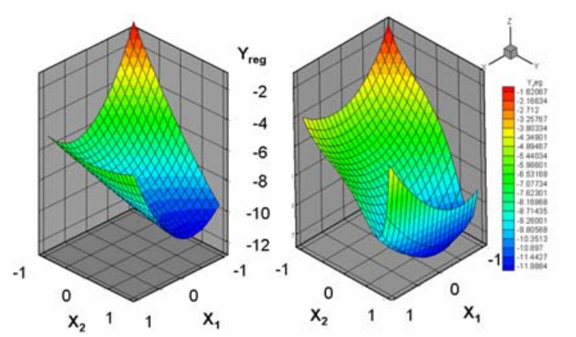
DESIGN APPLICATION USING RRSET

Numerical Example

To verify the effectiveness of the RRSET, a nonlinear numerical optimization problem with two design variables is performed.

$$\begin{aligned} &\textit{Minimize} \quad F = -3.26+3.0x_1^2 -2.02\cdot\exp(-x_2 \cdot x_1) +1.0/(-0.378+0.881x_2-0.825x_2^2) \\ &\textit{Subject to} \quad -1 \le \boldsymbol{x} \le 1 \end{aligned} \tag{3}$$

To construct the response surface model for this problem, seventeen experiment points are selected using the D-optimality Design, one of the Design of Experiment techniques.



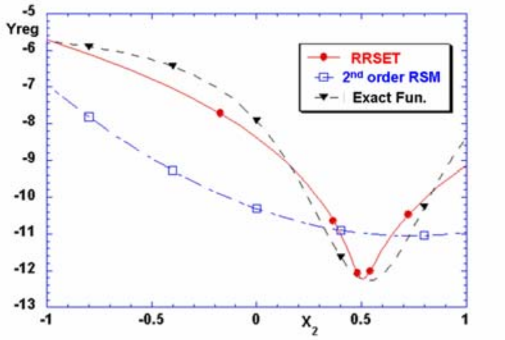


Figure 2. 2^{nd} order RSM(left) vs. RREST(right) Figure 3. Comparison of response surfaces

Figure 2 shows two response surfaces constructed by the ordinary second order polynomial (left) and RRSET(right). By implementing the RRSET, the accuracy of the RS can be greatly enhanced, especially near the optimum, which is our main concern. For the $R^2_{adj,}$ RRSET has 0.808, which shows improved accuracy of 24% compared with the case of conventional RSM with second order polynomial (R^2_{adj} of 0.615)

Design of a 2-Member Frame

Given 2-member frame design problem[9] is to minimize the total volume of the frames with given constraints. As design variables, the width, height, thickness of each frame are selected, and the allowable stress of each node is give as design constraint. The design formulation is given as follows;

$$\begin{aligned} &\textit{Minimize} \ V(x) = 2L(2d \cdot t + 2h \cdot t - 4h^2) \ \text{, where } 2.5 \le d, h \le 10,\ 0.1 \le t \le 1.0 \ \text{(Unit : in)} \\ &\textit{Subject to} \ g_1(x) = (\sigma_1^2 + 3\tau^2)^{1/2} \le 40{,}000,\ g_2(x) = (\sigma_2^2 + 3\tau^2)^{1/2} \le 40{,}000 \end{aligned} \tag{4}$$

For the optimization of this problem, three four experiment points are selected, hence three response surfaces, one for the objective function, two for the constraints are constructed. Table 1, and 2 show the constructed response surfaces and design results

Table1. 2^{nd} order RSM vs. RRSET(R^2_{adj})

Response	R^2_{adj} 2nd Order RSM	R^2_{adj} RRSET	%Improv.
Volume	1.0000	1.0000	-
E. Stress 1	0.9004	0.9620	6.84
E. Stress 2	0.8957	0.9564	6.34

Table 2. Results of optimization

Method	Opt. D.V.		Y*(pred)	Y*(exact)	%err.
Exact Solution	d*	7.798	-	703.916	-
	h*	10.000			
	t*	0.100			
2^{nd} Order RSM	d*	7.047	580.028	580.030	-17.600
	h*	7.654			
	t*	0.100			
RRSET	d*	7.890	707.616	707.620	0.526
	h*	10.000			
	t*	0.100			

CONCLUSIONS

In this study, RRSET has been proposed as a refined response surface method.

- To approximate highly nonlinear design space and to improve the confidence of response surface, the design space transformation technique using the stretching function is introduced.
- The sub-optimization for the most reliable regression model construction is formulated by selecting the adjusted R-squared as objective function and the stretching parameters as design variables. The simulated annealing algorithm (SA) is used to obtain global optimum parameters in the RS sub-optimization problem.
- System design process introducing RS sub-optimizer has been established. The tentative optimum points generated during the system design iteration process are introduced in the new experiment point to construct the response surface model in the next design iteration and system iteration with relatively small number is enough to get a very reliable response surface model.

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