A VERY SINGULAR SOLUTION FOR THE SLOW DIFFUSION EQUATION WITH NONLINEAR CONVCTION

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ABSTRACT

We study an existence and uniqueness of the very singular solution for a slow diffusion equation with nonlinear convection term defined on the half line:

$$u_t = (u^m)_{xx} + (u^q)_x \qquad in \quad Q = \mathbb{R}^+ \times \mathbb{R}^+$$

with homogeneous Neumann boundary condition, where $m > 1$ and $m < q < m + 1$. The solution we find is of the form

$$u(x,t) = t^{-\alpha} f(xt^{-\beta}) := t^{-\alpha} f(r), \quad r = xt^{-\beta},$$

where $\alpha = 1/(2q - m - 1)$, $\beta = (q - m)/(2q - m - 1)$, and f is the nontrivial, nonnegative solution of an nonlinear ordinary differential equation:

$$(f^m)'' + \beta r f' + \alpha f + (f^q)' = 0$$

with condition $f'(0) = 0$ and $\lim_{r\to\infty} r^{\alpha/\beta} f(r) = 0$.

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