BOUNDARY INTEGRAL EQUATION METHOD FOR PHOTONIC CRYSTAL FIBER

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ABSTRACT

A system of boundary integral equation for the calculation of the confinement loss of the photonic crystal fiber is derived from the two-dimensional Helmholtz equation using the Huygen's principle and the free space Green's function. The effective refractive indices, which will determine the confinement loss, for the air-silica and the photonic bandgap fibers are calculated as a function of wavelength.

INTRODUCTION

The photonic crystal fiber (PCF) [1] is an optical fiber, which has periodic holes surrounding the core of the fiber. Depending on the distribution of the refractive index, PCF can be categorized into the air-silica fiber the and photonic bandgap fiber (PBF). The traditional fiber guides light in the core, based on the Snell's law that limit the refractive index of the core higher than that of the cladding in order to have a Total Internal Reflection (TIR). In case of air-silica fiber, all the holes surrounding the core of the fiber are empty and thus the refractive index of the core is higher than that of surrounding holes. Therefore, the light is guided in the core according to the TIR. Figure 1 (a) is pictures of actual air-silica fiber used for the experiment. In case of PBF, all the surrounding holes are filled with a material with higher refractive index than that of the cladding (solid core PBF) [2] or air hole is placed in the core (hollow core PBF) [3]. As a consequence, the core of the fiber has a lower refractive index than that of the cladding and this violates the condition for the TIR. However, light is still guided via a constructive scattering from the surrounding holes with high refractive index.
In this abstract, boundary integral equations (BIE) are derived from the time harmonic Maxwell's equation for both types of the PCF and numerically solved for the effective refractive index, which determine the confinement loss of the fiber.

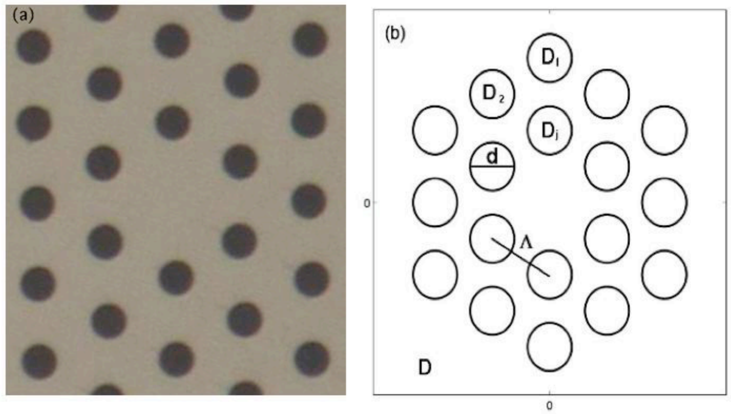

Figure 1. Photonic crystal fiber and computational domain

BOUNDARY INTEGRAL EQUATIONS AND NUMERICAL RESULTS

The governing equation is the time harmonic Maxwell equations and it can be reduced to the two-dimensional Helmholtz equation by assuming z dependance as $exp(i\beta z)$

$$\Delta u + k_j^2 u = 0, \tag{1}$$

where $k_j = (k^2 n_j^2 - \beta)^{(1/2)}$. Then, by a standard procedure, solution of the Helmholtz equation inside and outside of the microstructured holes can be written as

$$u(\hat{r}) = \int_{\partial D_j} [G_j(k_j||\hat{r} - r||)\frac{\partial u}{\partial n}(r) - \frac{\partial G_j}{\partial n}(k_j||\hat{r} - r||)u(r)]ds(r),\ for\ \hat{r} \in D_j, \tag{2}$$

$$u(\hat{r}) = \int_{\partial D_0} [-G_0(k_0||\hat{r} - r||)\frac{\partial u}{\partial n} + \frac{\partial G_0}{\partial n}(k_0||\hat{r} - r||)u(r)]ds(r),\ for\ \hat{r} \in R^2 \backslash (\cup D_j), \tag{3}$$

Equations (2) and (3) can be considered as Huygen's principle because the solutions away from the boundaries are written in terms of the field on the boundary. At last, in order to finish the formulation of BIE, let $\hat{r}$ approach the point on the boundary. Then, using the Plemelj formula, BIE can be derived as

$$u(\hat{r}) - 2\int_{\partial D_0} \frac{\partial G_0}{\partial n}(k_0||\hat{r} - r||)u(r)ds(r) + 2\int_{\partial D_0} G_0(k_0||\hat{r} - r||)\frac{\partial u}{\partial n}ds(r) = 0, \tag{4}$$

$$u(\hat{r}) + 2\int_{\partial D_j} \frac{\partial G_j}{\partial n}(k_j||\hat{r} - r||)u(r)ds(r) - 2\int_{\partial D_j} G_j(k_j||\hat{r} - r||)\frac{\partial u}{\partial n}(r)ds(r) = 0, \tag{5}$$

for $\hat{r} \in \partial D_0$. Left hand side of the equations (4) and (5) have unknown complex propagating constant β in the argument of the Green's function and its normal derivative. In order to have the nontrivial solutions, β that makes determinant of right hand side zero, are found with the Secant method. For the discretization of the integral operators, the method described in Colton and Kress's text [4] is used. For detailed numerical technique, see the reference [5]

The air-silica fiber and PBF are investigated with BIE and effective refractive index $n_{eff} = \beta/k_0$ are calculated as a function of wavelength λ. In the case of air-silica fiber, because the refractive index of the core is higher than that of the cladding, the imaginary part of effective refractive index ($Im(n_{eff})$) shows the gradual increment of the confinement loss as the wavelength increases due to a material dispersion of the silica like traditional optical fiber. Moreover, addition of the layer of holes reduces the confinement loss. In the case of PBF, $Im(n_{eff})$ has discontinuities which can be considered as loss peaks of the PBF. This can be explained using so called AntiResonant Reflecting optical Waveguide (ARROW) model [6] when the wavelength of the light is smaller than the size of holes. In this model, light is guided when it reflected from the high index Fabry-Perot like resonator and it leaks out of the core when the wavelength matches with the resonant condition of a single cylinder. The resonant condition of the single cylinder can be derived from the Bessel function and its cosine approximation as follows [7]

$$\lambda_m = \frac{2d(n_{high}^2 - n_{low}^2)^{1/2}}{m + 1/2}, \quad m = 1, 2, 3, \cdots . \tag{6}$$

The numerical results from BIE shows good agreement with the wavelength predicted by (6). At last, confinement loss of the PBF with refractive indices $n_1 = 1.44$ and $n_2 = 1.8$, length $L = 58.2\, cm$, diameter $d = 2.3\, \mu m$, and lattice constant $\Lambda = 5.4\, \mu m$ is measured and compared with simulation results.

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