CLOSED-FORM UPPER BOUNDS FOR THE OPTIMAL EXERCISE BOUNDARY OF AMERICAN PUT

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ABSTRACT

Kim (1990), Jacka (1991), and Carr, Jarrow, and Myneni (1992) showed that American option price is equal to the corresponding European option price plus an integral representing the early exercise premium. While the American option price has an explicit representation, the optimal exercise boundary is implicitly defined by a nonlinear integral equation. This article studies the properties of integral equations arising in the valuation of American options. Based on the properties of integral equations, this article also presents a series of closed form upper bounds for the optimal exercise boundary.

INTRODUCTION

Since most traded options are American type, the valuation of American options has become a primary concern in financial research. An important result in the problem of pricing American options is derived by Kim (1990), Jacka (1991), and Carr, Jarrow, and Myneni (1992). They derived a formula representing the early exercise premium of an American option as an integral which has the optimal exercise boundary in it. Several numerical methods for valuing American options, based on this integral representation formula, are proposed by Kim (1994), Huang, Subrahmanyam, and Yu (1996), Broadie and Detemple (1996), Ju (1998), AitSahlia and Lai (2001), and Kallast and Kivinukk (2003) among others. Kim (1994) proposed to approximate the integral in both the integral representation formula and in the integral equation by the trapezoidal rule. Huang, Subrahmanyam, and Yu (1996) proposed to approximate the integrands in both the integral representation formula and in the integral equation by piecewise constant functions. Ju (1998) proposed to approximate the boundary by a piecewise exponential function, for which the integral in the representation formula can be evaluated in closed form.

This integral representation formula requires the determination of the optimal exercise boundary for its implementation. To determine the optimal exercise boundary, we can apply "value-match" or "high-contact" condition to generate nonlinear integral equations. However, it is not possible to obtain the optimal exercise boundary in explicit form. So, analytic and numerical properties of the optimal exercise boundary have been investigated by Jacka (1991), Kim and Byun (1994) and Kim, Byun, and Lim (2004), Byun (2005), and Peskir (2005). Kim, Byun, and Lim (2004) studies the numerical properties of the optimal exercise boundary in the case of deterministic volatility function. Byun (2005) investigates the properties of the integral equation which must be satisfied by the optimal exercise boundary.

Asymptotic behavior of the optimal exercise boundary near the expiration time has also been investigated extensively in Kuske and Keller (1998), Bunch and Johnson (2000), Knessel (2001), Evans, Kuske, and Keller (2002) among others.

The purpose of this talk is to improve our understanding for the properties of integral equations arising in the valuation of American options. This enables us to develop a series of closed form upper bounds for the optimal exercise boundary.

Based on the properties of integral equations, we first introduce an intercept of a linear line and Black and Scholes European put valuation formula, denoted by B^a, and show that this intercept is a uniform upper bound for the optimal exercise boundary. We then derive a series of closed form upper bound for B^a and a series of closed form lower bound for B^a. It should be noted that an upper bound for B^a can also serve as an upper bound for the optimal exercise boundary.

The talk is organized as follows. First we review the properties of integral equations arising in the valuation of American options presented in Byun (2005). Based on the properties of integral equations,, and then we develop a series of closed form upper bounds for the optimal exercise boundary

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