Eigenvalue Problem for Singular One-Dimensional $p$-Laplacian and Its Applications

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ABSTRACT
In this talk, we establish a sequence of eigenvalues of singular $p$-Laplacian boundary value problem:
\[
\begin{cases}
-(\varphi_p(u'(t)))' = \lambda h(t) \varphi_p(u(t)) & \text{a.e. } (0, 1), \\
n(0) = u(1) = 0,
\end{cases}
\]
where $\varphi_p(x) = |x|^{p-2}x$, $p > 1$, $h \in L^1(0, 1)$ and $\lambda$ is a positive parameter. Next, we apply it to study global bifurcation phenomena of positive and sign-changing solutions for the following problem:
\[
\begin{cases}
-(\varphi_p(u'(t)))' = \lambda h(t) f(u(t)) & \text{a.e. } (0, 1), \\
n(0) = u(1) = 0.
\end{cases}
\]

INTRODUCTION
Consider the following singular boundary value problem
\[
\begin{cases}
-(\varphi_p(u'(t)))' = \lambda h(t) f(u(t)) & \text{a.e. } (0, 1), \\
n(0) = u(1) = 0,
\end{cases}
\]
where $\varphi_p(x) = |x|^{p-2}x$, $p > 1$, and $\lambda$ is a parameter. Here $h(t) \in L^1(0, 1)$ is a nonnegative measurable function on $(0, 1)$ that may be singular at $t = 0$ and/or $t = 1$, and the conditions on $f(u)$ will be given later.

By a solution of the problem $(P^p_\lambda)$, we mean a function $u(t) \in C[0, 1] \cap C^1(0, 1)$ which satisfies both the equation on $(0, 1)$ and the boundary condition in $(P^p_\lambda)$. Moreover $\varphi_p(u'(t))$ is locally absolutely continuous in $(0, 1)$ and the equality
\[-(\varphi_p(u'(t)))' = \lambda h(t) f(u(t))
\]
holds almost everywhere in $(0, 1)$.

The problem $(P^p_\lambda)$ was studied by several authors. Yang ([3]) showed the existence of the simple, isolated and positive eigenvalues which diverges to infinity for $(P^p_\lambda)$ with restriction on $h$ which is in $L^q(0, 1)$, $q > 1$, and $f(u) = \varphi_p(u)$, $p \geq 2$. Notice that he excluded the case $1 < p < 2$. In [2], Sánchez obtained the existence of multiple positive solutions for the problem $(P^p_\lambda)$ with a positive continuous function $h \in L^1(0, 1)$, and super and sublinear cases of $f$. We
denote the class
\[ A = \left\{ h \in C(0, 1) : \int_0^{\frac{1}{2}} \varphi^{-1} \left( \int_0^{\frac{1}{2}} h(r) dr \right) ds + \int_1^{1/2} \varphi^{-1} \left( \int_s^{1/2} h(r) dr \right) ds < \infty \right\}. \]

Clearly, we have \( L^1(0, 1) \subseteq A \). Agarwal et al. established the existence of multiple positive solutions for the problem \((P^p_\lambda)\) in [1] with \( h \in A \).

With help of the generalized Picone’s type identity and Prüfer transformation, we shall establish the sequence \( \{\lambda_k(p)\} \) of eigenvalues of the following problem:
\[
\begin{align*}
- (\varphi_p(u'(t)))' &= \lambda h(t) \varphi_p(u(t)) \quad \text{a.e. } (0, 1), \\
u(0) &= u(1) = 0.
\end{align*}
\]

We shall show the existence of alternatives of global bifurcation at \((\lambda_k(p), 0)\) in the sense of Rabinowitz and the unboundedness of a subcontinuum. Finally, we shall sketch the shape of a subcontinuum of positive solutions from \((\lambda_1, 0)\) and of sign-changing solutions from \((\lambda_k, 0), k \geq 2\).

REFERENCES

