An exact solution of location of a circle to a set of the weighted points

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ABSTRACT

We consider the minimax problems of location of the circle to a set of the weighted points with passing through the two fixed points. The problem can be applied to design of nano-scale circuits and facility location. We suggested the problem properties and the solution procedure. Finally, we resolved to find an exact solution of location of a circle with the suggested constraints and posted the numerical computation time with the number of points.

INTRODUCTION

Let \( P_j(x_j, y_j), j = 1, \cdots, n, \) be a given set of points on the plane. Also suppose that two additional points \( Q_1 \) and \( Q_2 \) are given which are distinct from \( P_j, j = 1, \cdots, n. \) We are interested in the constrained optimization problem of finding a circle that is closest to all points \( P_j, j = 1, \cdots, n, \) among all the circles that are constrained to pass through \( Q_1 \) and \( Q_2. \)

EQUATIONS

The closeness from a circle to a set of points is given by the weighted maximum distance from the circle to the points. Denote by \( X \) the center of a circle which passes through the two points \( Q_1 \) and \( Q_2, \) and by \( D_w(X) \) the \( n \)-dimensional vector

\[
D_w(X) = (w_1d_1(X), \cdots, w_n d_n(X)),
\]

where \( d_j(X) = |P_j - X| - |Q_1 - X|, \) and the weight \( w_j \in (0, 1), j = 1, \cdots, n. \)

Set the objective function \( J_p(X) = \|D_w(X)\|_p, \) where \( \| \cdot \|_p \) denotes the \( \ell^p \)-norm for \( 1 \leq p \leq \infty. \)

Our constrained optimization problem is then formulated as an unconstrained minimax problem of finding \( X^* \) such that

\[
J_{\infty}(X^*) = \min_{X \in \mathbb{R}^2} J_{\infty}(X).
\]

See the references [1,2,3,4,5,6,7,8,9,10,11,12,14,13,5,6,7,8,9,11,12,14]
CONCLUSIONS

In this paper we restrict to the optimization problem of finding a circle which passes through two given points. We propose a systematic approach to resolve this minimax problems, obtaining an exact solution algorithm which is very fast.

REFERENCES