FAST NONLOCAL REGULARIZATION METHOD FOR IMAGE RESTORATION

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ABSTRACT

In this paper, we propose an alternating minimization algorithm to solve nonlocal total variation based minimization problems. Because nonlocal total variation are designed based on self-similarity of images, it is very useful for various image restoration problems. Recently, several efficient optimization methods are developed to solve the Nonlocal TV minimization problem\([4,6]\). These methods are efficient but slow to handle deblurring problem. In this paper, we show how to efficiently enhance blurred image with alternating minimization algorithm\([7]\) and Bregman operator splitting method\([6]\).

NONLOCAL TOTAL VARIATION

Image restoration can be formulated as an inverse problem. Let \(\Omega \subset \mathbb{R}^2, x \in \Omega\) and the given observed image \(f : \Omega \rightarrow \mathbb{R}\). The objective is to find the unknown true image \(u \in \Omega\) from an observed image \(f\) defined by

\[
f = Au + \eta,
\]

where \(\eta \sim \mathcal{N}(0, \sigma^2)\) and \(A\) is a bounded linear operator, typically a convolution operator in image deblurring, a projection in image inpainting and the identity in image denoising. Since inverse problems are typically ill-posed, it is common to use a regularization technique to make them well-posed. The total variation regularization has been popular ever since its introduction by Rubin et al. \([1]\). The advantage of using TV regularization is to preserve edges due to its linear penalty on differences between adjacent pixels.

Recently, nonlocal means\([2]\) are proposed as an efficient denoising model. The basic idea is simple. By adaptively averaging the other pixels with similar structure to the current pixel, we could dramatically increase image quality. The definitions and notations of the nonlocal operators in \([3]\) are used to define nonlocal total variation. Assume \(w : \Omega \times \Omega \rightarrow \mathbb{R}^+ \cup \{0\}\) is a symmetric weight function. For examples,

\[
w(x, y) = e^{-||F_f(x) - F_f(y)||^2_2/h^2},
\]

where \(F_f(x) = f(x) \in B(x)\) and \(B(x)\) is a \(a \times b\) size patch. \(h\) is the scaling parameter which determines the similarity between different patches. The nonlocal gradient operator, \(\nabla_{NL}u(x)\) is defined as the vector of all partial derivatives:

\[
\nabla_{NL}u(x, y) = (u(y) - u(x))\sqrt{w(x, y)}, \quad \text{for all} \ y \in \Omega,
\]
where \( w(x, y) \) is the weight function between \( x \) and \( y \). The nonlocal divergence operator with the nonlocal gradient (generalized divergence theorem):

\[
< \nabla_{NL} u, p > = -< u, \text{div}_{NL} p >,
\]

\( \forall u : \Omega \rightarrow \mathbb{R}, \forall p : \Omega \times \Omega \rightarrow \mathbb{R} \), which defines the NL divergence of the NL vector \( p : \Omega \times \Omega \rightarrow \mathbb{R} \) at \( x \in \Omega \):

\[
\text{div}_{NL} p(x) = \int_{\Omega} (p(x, y) - p(y, x)) \sqrt{w(x, y)} dy : \Omega \rightarrow \mathbb{R}.
\]

The nonlocal Laplacian (graph Laplacian) is defined in the following

\[
\Delta_{NL} u(x) = \frac{1}{2} \text{div}_{NL}(\nabla_{NL} u(x)) = \int_{\Omega} (u(y) - u(x)) w(x, y) dy.
\]

We could define nonlocal total variation functional by

\[
TV_{NL}(u) = \int_{\Omega} |\nabla_{NL} u| dx = \int_{\Omega} \sqrt{\int_{\Omega} (u(z) - u(x))^2 w(x, z) dz} dx
\]

To find true image \( u \), we could use unconstrained variational model

\[
\arg\min_u \mu TV_{NL}(u) + \frac{1}{2} ||Au - f||^2
\]

where

\[
TV_{NL}(u) = \int |\nabla_{NL} u(x)| dx = \int_{\Omega} \sqrt{\int_{\Omega} (u(z) - u(x))^2 w(x, z) dz} dx
\]

Note that if \( A = I \) then (2) become the nonlocal ROF denoising model.

**PROPOSED ALGORITHM**

To find solution of the nonlocal total variation inverse problem (2), Zhang et.al.[6] proposed Bregman operator splitting method with split Bregman iteration. Split Bregman[5] is known to be very powerful and speedy but we need to solve nonlocal poisson equation. Instead of split Bregman, we propose to use AMA(Alternating Minimization Algorithm) in [7]. Since AMA does not need to solve complicated nonlocal poisson equation, it is more faster then split Bregman and the convergence is also proved[7]. The following is the proposed algorithm:

\[
v^{k+1} = u^k - \delta A^T (Au^k - f^k)
\]

\[
u^{k+1} = \max_b \min_{u,d} (\mu |d|^1 + \frac{1}{2\delta} ||u - v^{k+1}||^2 + < b, d - \nabla u >)
\]

\[
f^{k+1} = f^k + f - Au^{k+1}
\]

where (5) could be solved by the following iteration:

\[
u^{k+1} = \lim_{l \rightarrow \infty} u_l \begin{cases} u_{l+1} = \arg\min_u (\frac{1}{2\delta} ||u - v^{k+1}||^2 + < b_l, d_l - \nabla u >) \\ d_{l+1} = \arg\min_d (\mu |d|^1 + \frac{1}{2\delta} ||d - \nabla u_{l+1} + b_l||^2 + \frac{\lambda}{\delta} ||d - \nabla u_{l+1} + b_l||^2) \\ b_{l+1} = b_l + \lambda (d_{l+1} - \nabla u_{l+1}). \end{cases}
\]
For simplicity, we skip weight update scheme. For detail, see [6,4].

**EXPERIMENTS**

In the section we compare the proposed method with BOS and split Bregman for nonlocal TV model and the local TV model for deblurring problem. We gave test results in Figure 1. Here we use 10 best neighbor and the search window size is $21 \times 21$ and the patch size is $5 \times 5$. For the local TV deblurring, we also use BOS and split Bregman scheme. The added noise is gaussian noise $\sim \mathcal{N}(0, 3^2)$. The first image in Fig. 1 is original image and second image is blurry and noisy image. Third image is the result of local TV. Fourth image is the result of BOS and split Bregman for nonlocal TV. Fifth image is the result of the proposed method, that is, BOS and AMA.

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Figure 1. The first image is original image and the second image is contaminated image. Third is local TV image with (PSNR, CPU time) = (24.6dB, 11sec). Fourth is BOS and split Bregman with (24.8dB, 368sec). Fifth is BOS and AMA with (25.1dB, 221sec).