STRONGLY NONLINEAR INTERNAL LONG WAVE MODEL

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ABSTRACT

Internal waves propagate in the oceans where denser, colder and saltier deep waters meet warmer, fresher and less dense upper waters. These waves evolve due to the bottom topography, currents, and others, and they may contribute significantly to mixing and transpoting energy in the ocean. Packets of large amplitude internal waves have been observed in many coastal regions around the world. However, understanding dynamics of a large amplitude wave is difficult comparing to that of a small amplitude wave because of its strong nonlinearity. Although the generation and evolution of internal waves over variable topography are governed by three dimensional Navier-Stokes equations, they are computationally too expensive to describe the evolution of such strongly nonlinear long waves over a large area. Hence, approximated models derived from the Navier-Stokes equations or Euler equations have been developed in many ways for real applications. Each model relies on an assumption regarding the relative importance of the nonlinear and dispersive terms in the asymptotic expansion procedure. The ratio of wave amplitude to characteristic vertical length scale such as the thickness of the upper mixed layer is typically $O(1)$ and most theoretical models developed for weakly nonlinear waves are often inapplicable [4].

It has been shown that the strongly nonlinear long-wave models [3] for two-layer system obtained without the classical small-amplitude assumption is a good approximation to the Euler equations even for the strongly nonlinear regime as long as its traveling wave solutions are concerned [1]. The model also shows excellent agreement with laboratory experiments for the shallow and deep water configurations, respectively. Despite their success in describing traveling solitary wave solutions, the strongly nonlinear models have not been used much to solve time-dependent internal wave problems. In particular, a major difficulty in solving numerically the strongly nonlinear model for the shallow configuration is that, regardless of the wave amplitude, the model suffers from the local Kelvin-Helmoltz (KH) instability associated with the velocity discontinuity across the interface, as shown in [5].

Understanding limitation of the model, the evolution of a traveling wave and collision of traveling waves have been successfully simulated in the regime of stable parameters. It is also confirmed that the short-wavelength instability in numerical simulation is consistent with the
stability analysis shown in [5]. However, finite amplitude solitary waves generated in the laboratory experiments are found to be stable unless the wave amplitude is very close to the maximum wave amplitude. This discrepancy between the model from the inviscid assumption and the laboratory experiment may be overcome by deriving a viscous model, but it is not a trivial task. A numerical filter is adopted for suppress undesirable short wave instability without affecting the long wave behaviour and the propagation of traveling wave is simulated even with parameters in unstable regimes based on the linear stability analysis [6]. While a low-pass numerical filter is found to be effective to eliminate the KH instability, it is less useful for more general time-dependent problems since the choice of the cut-off wave number is arbitrary.

A regularized model for strongly nonlinear internal waves which uses the velocities of top and bottom boundaries is recently proposed [2]. It is asymptotically consistent with the original strongly nonlinear model, but it has a different dispersive behavior for short waves. For a wide range of depth and density ratios, it is found that the critical wave amplitude for stability is close to the maximum amplitude. It is solved numerically using finite difference method and its numerical solutions support the result of the linear stability analysis.

REFERENCES