SOME EXAMPLES OF QUASI-ARMENDARIZ RINGS

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ABSTRACT. In [12], McCoy proved that if \( R \) is a commutative ring, then whenever \( g(x) \) is a zero-divisor in \( R[x] \), there exists a nonzero \( c \in R \) such that \( cg(x) = 0 \). In this paper, first we extend this result to monoid rings. Then for a monoid \( M \), we give some examples of \( M \)-quasi-Armendariz rings which are a generalization of quasi-Armendariz rings. Every reduced ring is \( M \)-quasi-Armendariz for any unique product monoid \( M \) and any strictly totally ordered monoid \((M, \leq)\). Also \( T_k(R) \) is \( M \)-quasi-Armendariz when \( R \) is reduced and \( M \)-Armendariz.

1. Introduction

Throughout this paper \( R \) denotes an associative ring with identity. Rege and Chhawchharia [15] introduced the notion of an Armendariz ring. A ring \( R \) is called Armendariz if whenever polynomials \( f(x) = a_0 + a_1x + \cdots + a_nx^n, g(x) = b_0 + b_1x + \cdots + b_mx^m \in R[x] \) satisfy \( f(x)g(x) = 0 \), then \( a_ib_j = 0 \) for each \( i, j \).

Some properties of Armendariz rings have been studied in Rege and Chhawchharia [15], Armendariz [1], Anderson and Camillo [2], and Kim and Lee [9]. According to Hirano [5], a ring \( R \) is called to be quasi-Armendariz if whenever polynomials \( f(x) = a_0 + a_1x + \cdots + a_nx^n, g(x) = b_0 + b_1x + \cdots + b_mx^m \in R[x] \) satisfy \( f(x)R[x]g(x) = 0 \), then \( a_iRb_j = 0 \) for each \( i, j \). In [5], Hirano studied some properties of quasi-Armendariz rings. In [17], Zhongkui studied a generalization of Armendariz rings, which is called \( M \)-Armendariz rings, where \( M \) is a monoid. A ring \( R \) is called \( M \)-Armendariz if whenever \( \alpha = a_1g_1 + \cdots + a_ng_n, \beta = b_1h_1 + \cdots + b_nh_n \in R[M] \), with \( g_i, h_j \in M \) satisfy \( \alpha \beta = 0 \), then \( a_ib_j = 0 \) for each \( i, j \). Recall that a monoid \( M \) is called a u.p.-monoid (unique product monoid) if for any two nonempty finite subset \( A, B \subseteq M \) there exists an element \( g \in M \) uniquely presented in the form \( ab \) where \( a \in A \) and \( b \in B \). The class of u.p.-monoid is quite large and important (see [3, 13, 14]). For example, this class includes the right or left ordered monoids, submonoids of a free group, and torsion-free nilpotent groups. Every u.p.-monoid \( M \) has no non-unity element of finite order. For \( \alpha = a_1g_1 + \cdots + a_ng_n \in R[M] \) with \( a_i \neq 0 \) for each \( i \), \( \text{length}(\alpha) \) is defined to be \( n - k + 1 \).

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In this paper, for a monoid $M$, we give some examples of $M$-quasi-Armendariz rings which are a generalization of quasi-Armendariz rings. Every reduced ring is $M$-quasi-Armendariz for any unique product monoid $M$ and any strictly totally ordered monoid $(M, \leq)$. Also, $T_2(R)$ is $M$-quasi-Armendariz when $R$ is reduced and $M$-Armendariz.

2. Some examples of quasi-Armendariz rings

McCoy [12] proved that if $R$ is a commutative ring, then whenever $g(x)$ is a zero-divisor in $R[x]$ there exists a nonzero element $c \in R$ such that $cg(x) = 0$. Hirano [5] extend this result to a non commutative ring as follows. If $r_{R[x]}(f(x)R[x]) \neq 0$ for $f(x) \in R[x]$, then $r_{R[x]}(\alpha R[x]) \cap R \neq 0$. We shall generalize this result to monoid rings as follows:

**Theorem 2.1.** Let $M$ be a u.p.-monoid or $(M, \leq)$ be a totally ordered monoid. Let $\alpha$ be an element of $R[M]$. If $r_{R[M]}(\alpha R[M]) \neq 0$ then $r_{R[M]}(\alpha R[M]) \cap R \neq 0$.

**Proof.** We prove it for a u.p.-monoid. The other case is similar. Let $\alpha = a_1g_1 + \cdots + a_ng_n$. If $n = 1$, then assertion is clear. Let $n \geq 2$. Assume that $\beta = b_1h_1 + \cdots + b_nh_n \in R[M]$ be a nonzero element of minimal length in $r_{R[M]}(\alpha R[M])$.

Since $(\alpha R[M]\beta) = 0$, $\alpha R\beta = 0$. Since $M$ is a u.p.-monoid, there exists $i, j$ with $1 \leq i \leq n$, $1 \leq j \leq m$ such that $a_ih_j$ is uniquely presented by considering two subsets $A = \{g_1, \ldots, g_s\}, B = \{h_1, \ldots, h_t\}$ of $M$. Thus $a_icb_jgh_j = 0$ for each $c \in R$ and hence $a_iRb_j = 0$. Thus $0 = \alpha(R[M]a_iR[M])(b_1h_1 + \cdots + b_nh_n) = \alpha R[M](a_iR[M](b_1h_1 + \cdots + b_{i-1}h_{i-1} + b_{i+1}h_{i+1} + \cdots + b_nh_n)).$ By hypothesis, $a_iR(b_1h_1 + \cdots + b_{i-1}h_{i-1} + b_{i+1}h_{i+1} + \cdots + b_nh_n) = 0$. Therefore $a_iRb_j = 0$ for each $1 \leq t \leq m$. Hence $(a_ig_1 + \cdots + a_{i-1}g_{i-1} + a_{i+1}g_{i+1} + \cdots + a_ng_n)(R[M]\beta) = 0$. Since $M$ is u.p.-monoid, there exist $r, s$ with $r \in \{1, \ldots, i-1, i+1, \ldots, n\}$ and $s \in \{1, \ldots, m\}$ such that $g_rh_s$ is uniquely presented by considering two subsets $A = \{g_1, \ldots, g_{r-1}, g_{r+1}, \ldots, g_n\}, B = \{h_1, \ldots, h_m\}$ of $M$. Thus $a_rcb_jgh_s = 0$ for each $c \in R$ and hence $a_iRb_s = 0$. Thus $0 = \alpha(R[M]a_iR[M])(b_1h_1 + \cdots + b_nh_n) = \alpha R[M](a_iR[M](b_1h_1 + \cdots + b_{r-1}h_{r-1} + b_{r+1}h_{r+1} + \cdots + b_nh_n)).$ By hypothesis, $a_iR(b_1h_1 + \cdots + b_{r-1}h_{r-1} + b_{r+1}h_{r+1} + \cdots + b_nh_n) = 0$. Therefore $a_iRb_t = 0$ for each $1 \leq t \leq m$. Repeating this process, we obtain $a_iRb_m = 0$ for each $1 \leq i \leq n$. Hence $b_m \in r_{R[M]}(\alpha R[M])$. Therefore $r_{R[M]}(\alpha R[M]) \cap R \neq 0$.

**Corollary 2.2** ([5], Theorem 2.2). Let $f(x)$ be an element of $R[x]$. If $r_{R[x]}(f(x)R[x]) \neq 0$, then $r_{R[x]}(f(x)R[x]) \cap R \neq 0$.

We investigate a generalization of quasi-Armendariz rings which we call an $M$-quasi-Armendariz ring.

**Definition 2.3.** Let $M$ be a monoid. We say that $R$ is an $M$-quasi-Armendariz if $\alpha = a_1g_1 + \cdots + a_ng_n, \beta = b_1h_1 + \cdots + b_nh_n \in R[M]$ satisfy $\alpha R[M]\beta = 0$, then $a_iRb_j = 0$ for each $i, j$. 
If \( M = (\mathbb{N} \cup \{0\}) \), then \( R \) is \( M \)-quasi-Armendariz if and only if \( R \) is quasi-Armendariz. If \( R \) is reduced and \( M \)-Armendariz, then \( R \) is \( M \)-quasi-Armendariz.

**Proposition 2.4.** Let \( M \) be a u.p.-monoid and \( R \) be a reduced ring. Then \( R \) is \( M \)-quasi-Armendariz.

**Proof.** Let \( \alpha = a_1g_1 + \cdots + a_ng_n \) and \( \beta = b_1h_1 + \cdots + b_mh_m \in R[M] \) be such that \( \alpha R[M] \beta = 0 \). We show that \( a_iRb_j = 0 \) for each \( i, j \). We proceed by induction on \( m \). It is clear for \( m = 1 \). Since \( M \) is a u.p.-monoid, there exists \( i, j \) with \( 1 \leq i \leq n \) and \( 1 \leq j \leq m \) such that \( g_ih_j \) is uniquely present by considering two subsets \( A = \{g_1, \ldots, g_n\} \) and \( B = \{h_1, \ldots, h_m\} \) of \( M \). Thus \( a_iRb_j = 0 \) and that \( a_iRb_j = 0 \). Thus \( 0 = (a_1g_1 + \cdots + a_ng_n)R[M](a_1b_1 + \cdots + b_mh_m) = (a_1g_1 + \cdots + a_ng_n)R[M](a_1b_1h_1 + \cdots + a_nb_mh_m) \). By induction, it follows that \( a_iRb_q = 0 \) for \( q = 1, \ldots, m \). Then \( a_iRb_q = 0 \), for each \( q = 1, \ldots, m \), since \( R \) is reduced. Thus \( (a_1g_1 + \cdots + a_{i-1}g_{i-1} + a_{i+1}g_{i+1} + a_ng_n)R[M](b_1h_1 + \cdots + b_mh_m) = 0 \). Continuing this procedure yield \( a_iRb_j = 0 \) for each \( 1 \leq i \leq n \), \( 1 \leq j \leq m \). Therefore \( R \) is \( M \)-quasi-Armendariz.

Let \((M, \leq)\) be an ordered monoid. If for any \( g_1, g_2, h \in M \), \( g_1 < g_2 \) implies \( g_1h < g_2h \) and \( hg_1 < hg_2 \), then \((M, \leq)\) is called a strictly ordered monoid.

**Proposition 2.5.** Let \( M \) be a strictly totally ordered monoid and \( R \) a reduced ring. Then \( R \) is \( M \)-quasi-Armendariz.

**Proof.** Let \( \alpha = a_1g_1 + \cdots + a_ng_n \) and \( \beta = b_1h_1 + \cdots + b_mh_m \in R[M] \) be such that \( \alpha R[M] \beta = 0 \) and \( g_i < \cdots < g_n \), \( h_1 < \cdots < h_m \). We use transfinite induction on the strictly totally ordered set \((M, \leq)\) to show that \( a_iRb_j = 0 \) for each \( i, j \).

If there exist \( 1 \leq i \leq n \) and \( 1 \leq j \leq m \) such that \( g_ih_j = g_ih_1 \), then \( g_i \leq g_j \) and \( h_1 \leq h_j \). If \( g_i < g_j \), then \( g_ih_1 < g_jh_1 \leq g_jh_j = g_ih_1 \) a contradiction. Thus \( g_i = g_j \). Similarly, \( h_1 = h_j \). Hence \( a_iRb_1 = 0 \). Now suppose that \( \omega \in M \) is such that for any \( g_i \) and \( h_j \) with \( g_ih_j < \omega \), \( a_iRb_j = 0 \). We will show that \( a_iRb_j = 0 \) for any \( g_i \) and \( h_j \) with \( g_ih_j = \omega \). Set \( X = \{(g_i, h_j) | g_ih_j = \omega \} \). Then \( X \) is a finite set. We write \( X \) as \( \{(g_i, h_j) | t = 1, \ldots, k \} \) such that \( g_i < \cdots < g_n \).

Since \( M \) is cancellative, \( g_i = g_{i_2} \) and \( g_ih_j = g_{i_2}h_j = \omega \) imply \( h_{j_1} = h_{j_2} \).

Since \( \leq \) is a strict order, \( g_i < g_{i_2} \) and \( g_ih_{j_1} = g_{i_2}h_{j_2} = \omega \) imply \( h_{j_2} < h_{j_1} \).

Thus we have \( h_{j_2} < \cdots < h_{j_2} < h_{j_1} \). Now

\[
\sum_{(g_i, h_j) \in X} a_ib_j = \sum_{t=1}^{k} a_{i_t}b_{j_t} = 0.
\]

For any \( t \geq 2 \), \( g_{i_t}h_j < g_{i_{t-1}}h_j = \omega \), and thus, by induction hypothesis, we have \( A_{i_t}Rb_{j_t} = 0 \) for each \( t = 2, \ldots, k \). By multiplying \( a_{i_t} \) to Eq.(1), from the left hand-side, we have \( a_{i_t}a_{i_{t-1}}b_{j_t} = 0 \). Since \( R \) is reduced, we have \( a_{i_t}b_{j_t} = 0 \). Now
Eq. (1), becomes

\[ \sum_{i=2}^{k} a_i b_i = 0. \]

By multiplying \( a_i \) to Eq. (2), from the left hand-side, we obtain \( a_i b_j = 0 \) by the same way as above. Continuing this process, we can prove \( a_i b_j = 0 \) for any \( i, j \) with \( g_i h_j = \omega \). Therefore, by transfinite induction, \( a_i b_j = 0 \) for any \( i, j \). Thus \( a_i b_j = 0 \) for any \( i, j \), since \( R \) is reduced. Therefore \( R \) is \( M \)-quasi-Armendariz.

**Corollary 2.6.** Let \( R \) be a reduced ring. Then \( R \) is \( Z \)-quasi-Armendariz, that is for any \( \alpha = a_{-m} x^{-m} + \cdots + a_0 x^0, \beta = b_{-n} x^{-n} + \cdots + b_0 x^0 \in R[x, x^{-1}] \), if \( \alpha R[x, x^{-1}] \beta = 0 \), then \( a_i R b_j = 0 \) for each \( i, j \).

**Proposition 2.7.** Let \( M \) be a u.p.-monoid or \((M, \leq)\) be a strictly totally ordered monoid and \( I \) an ideal of \( R \). If \( I \) is a reduced and \( R/I \) is \( M \)-quasi-Armendariz, then \( R \) is \( M \)-quasi-Armendariz.

**Proof.** We prove it for u.p.-monoid. The other case is similar. Let \( \alpha = a_1 g_1 + \cdots + a_n g_n \) and \( \beta = b_1 h_1 + \cdots + b_m h_m \in R[M] \) be such that \( \alpha R[M] \beta = 0 \). Since \( M \) is a u.p.-monoid, there exists \( i, j \) with \( 1 \leq i \leq n \) and \( 1 \leq j \leq m \) such that \( g_i h_j \) is uniquely present by considering two subsets \( A = \{ g_1, \ldots, g_n \} \) and \( B = \{ h_1, \ldots, h_m \} \) of \( M \). Thus \( a_i R b_j h_j = 0 \) and that \( a_i R b_j = 0 \). Thus

\[
0 = (a_1 g_1 + \cdots + a_n g_n) R[M] a_i (b_1 h_1 + \cdots + b_m h_m) \\
= (a_1 g_1 + \cdots + a_n g_n) R[M] (a_i b_1 h_1 + \cdots + a_i b_j h_j + \cdots + a_i b_m h_m).
\]

Thus, by induction hypothesis, we have \( a_i R a_j b_j = 0 \) for each \( j = 1, \ldots, m \). Note that in \((R/I)[M], (\pi_1 g_1 + \cdots + \pi_n g_n) R/I (b_1 h_1 + \cdots + b_m h_m) = 0\). Thus we have \( a_i R b_j \subseteq I \) for each \( i, j \), since \( R/I \) is \( M \)-quasi-Armendariz. Hence \( (a_i b_j)^2 = 0 \) and that \( a_i b_j = 0 \) for \( j = 1, \ldots, m \), since \( I \) is reduced and \( a_i b_j \in I \). Thus \( 0 = (a_1 g_1 + \cdots + a_{i-1} g_{i-1} + a_{i+1} g_{i+1} + \cdots + a_n g_n) R[M] (b_1 h_1 + \cdots + b_m h_m) = 0 \). Therefore, by induction on \( m+n \), we have \( a_i R b_j = 0 \) for each \( i, j \). Consequently \( R \) is \( M \)-quasi-Armendariz.

Recall that a monoid \( M \) is called torsion-free if the following property holds: if \( g, h \in M \) and \( k \geq 1 \) are such that \( g^k = h^k \), then \( g = h \).

**Corollary 2.8.** Let \( M \) be a commutative, cancellative and torsion-free monoid. If one of the following conditions holds, then \( R \) is \( M \)-quasi-Armendariz:

1. \( R \) is reduced.
2. \( R/I \) is \( M \)-quasi-Armendariz for some ideal \( I \) of \( R \) and \( I \) is reduced.

**Proof.** If \( M \) is commutative, cancellative and torsion-free, then by [16] there exists a compatible strict total ordered \( \leq \) on \( M \). Now the results follows from Proposition 2.5 and 2.7.
Proposition 2.9. Let $M$ be a cyclic group of order $n \geq 2$ and $R$ a ring with $0 \neq 1$. Then $R$ is not $M$-quasi-Armendariz.

Proof. Suppose that $M = \{e, g, g^2, \ldots, g^{n-1}\}$. Let $\alpha = 1 + g + g^2 + \cdots + g^{n-1}$ and $\beta = 1 + (-1)g$. Then $\alpha \beta = 0$ for each $c \in R$ and that $\alpha R[M] \beta = 0$. Thus $R$ is not $M$-quasi-Armendariz. \hfill \Box

Example 2.10. Let $R$ be an $M$-Armendariz and reduced ring. Let

$$T_4(R) = \left\{ \begin{pmatrix} a & a_{12} & a_{13} & a_{14} \\ 0 & a & a_{23} & a_{24} \\ 0 & 0 & a & a_{34} \\ 0 & 0 & 0 & a \end{pmatrix} \mid a, a_{ij} \in R \right\}.$$

Then $T_4(R)$ is $M$-quasi-Armendariz. It is easy to see that there exists an isomorphism of rings $T_4(R)[M] \rightarrow T_4(R[M])$ defined by:

$$\begin{pmatrix} a^k & a_{12}^k & a_{13}^k & a_{14}^k \\ 0 & a^k & a_{23}^k & a_{24}^k \\ 0 & 0 & a^k & a_{34}^k \\ 0 & 0 & 0 & a^k \end{pmatrix} g_k \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 \\ a \sum_{k=1}^{s} a_{12}^k g_k & a_{13}^k g_k & a_{14}^k g_k & \cdots \\ 0 & a \sum_{k=1}^{s} a_{23}^k g_k & a_{24}^k g_k & \cdots \\ 0 & 0 & a \sum_{k=1}^{s} a_{34}^k g_k & \cdots \end{pmatrix} = \begin{pmatrix} \alpha & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ 0 & \alpha & \alpha_{23} & \alpha_{24} \\ 0 & 0 & \alpha & \alpha_{34} \\ 0 & 0 & 0 & \alpha \end{pmatrix}.$$
with $a_{kk}^i = a_{kk}^t$ and $b_{kk}^i = b_{kk}^t$ for each $i, j, k, t$. Let

$$X = \left( \begin{array}{cccc}
\sum_{i=1}^s a_{11}^i g_i & \sum_{i=1}^s a_{12}^i g_i & \sum_{i=1}^s a_{13}^i g_i & \sum_{i=1}^s a_{14}^i g_i \\
0 & \sum_{i=1}^s a_{22}^i g_i & \sum_{i=1}^s a_{23}^i g_i & \sum_{i=1}^s a_{24}^i g_i \\
0 & 0 & \sum_{i=1}^s a_{33}^i g_i & \sum_{i=1}^s a_{34}^i g_i \\
0 & 0 & 0 & \sum_{i=1}^s a_{44}^i g_i 
\end{array} \right),$$

and

$$Y = \left( \begin{array}{cccc}
\sum_{i=1}^s b_{11}^i g_i & \sum_{i=1}^s b_{12}^i g_i & \sum_{i=1}^s b_{13}^i g_i & \sum_{i=1}^s b_{14}^i g_i \\
0 & \sum_{i=1}^s b_{22}^i g_i & \sum_{i=1}^s b_{23}^i g_i & \sum_{i=1}^s b_{24}^i g_i \\
0 & 0 & \sum_{i=1}^s b_{33}^i g_i & \sum_{i=1}^s b_{34}^i g_i \\
0 & 0 & 0 & \sum_{i=1}^s b_{44}^i g_i 
\end{array} \right).$$

Then we have $XAY = 0$ for each $A \in T_s(R[M])$. We show that $\alpha_{ij} \beta_{jk} = 0$ for each $i = 1, 2, 3, 4$, $j = 1, 2, 3, 4$ and $k = 1, 2, 3, 4$. Since $XT_s(R[M])Y = 0$, we have

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\
0 & \alpha_{22} & \alpha_{23} \\
0 & 0 & \alpha_{33} \end{pmatrix} \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\
0 & \beta_{22} & \beta_{23} \\
0 & 0 & \beta_{33} \end{pmatrix} = 0,$$

and

$$\begin{pmatrix} \alpha_{22} & \alpha_{23} & \alpha_{24} \\
0 & \alpha_{33} & \alpha_{34} \\
0 & 0 & \alpha_{44} \end{pmatrix} \begin{pmatrix} \beta_{22} & \beta_{23} & \beta_{24} \\
0 & \beta_{33} & \beta_{34} \\
0 & 0 & \beta_{44} \end{pmatrix} = 0.$$

By ([15], Proposition 1.7), $\alpha_{11} \beta_{11} = \alpha_{11} \beta_{12} = \alpha_{11} \beta_{13} = \alpha_{22} \beta_{22} = \alpha_{12} \beta_{23} = \alpha_{13} \beta_{33} = \alpha_{23} \beta_{33} = 0$ and $\alpha_{22} \beta_{23} = \alpha_{22} \beta_{24} = \alpha_{23} \beta_{34} = \alpha_{24} \beta_{44} = \alpha_{33} \beta_{34} = \alpha_{34} \beta_{44} = 0$. Since $XT_s(R[M])Y = 0$, we have $\alpha_{11} \beta_{14} + \alpha_{12} \beta_{24} + \alpha_{13} \beta_{34} + \alpha_{14} \beta_{44} = 0$. Since $R[M]$ is reduced and $\alpha_{11} = \alpha_{1j}$ and $\alpha_{ni} \beta_{im} = 0$ for each $i = 2, \ldots, n$, if we multiply this equation on the left side by $\alpha_{11}$, then $\alpha_{11} \alpha_{11} \beta_{14} = 0$ and that $\alpha_{11} \beta_{14} = 0$. Hence $\alpha_{12} \beta_{24} + \alpha_{13} \beta_{34} + \alpha_{14} \beta_{44} = 0$. Also if we multiply this equation on the right side by $\beta_{44}$, then $\alpha_{14} \beta_{44} \beta_{44} = 0$. Thus $\alpha_{14} \beta_{44} = 0$, since $\beta_{44} = \beta_{jj}$ for each $j$ and $R[M]$ is reduced. Thus
\[ \alpha_{12}\beta_{24} + \alpha_{13}\beta_{34} = 0. \] Hence

\[
X \begin{pmatrix}
\alpha_{12} & 0 & 0 & a \\
0 & \alpha_{12} & \alpha_{13} & 0 \\
0 & 0 & \alpha_{12} & 0 \\
0 & 0 & 0 & \alpha_{12}
\end{pmatrix}
Y
= \begin{pmatrix}
\alpha_{11}\alpha_{12}\beta_{11} & \cdots & \alpha_{13}\alpha_{12}\beta_{34} \\
0 & \alpha_{22}\alpha_{12}\beta_{22} & \cdots & \alpha_{33}\alpha_{12}\beta_{33} \\
0 & 0 & \alpha_{13}\alpha_{12}\beta_{33} & \cdots \\
0 & 0 & 0 & \alpha_{nn}\alpha_{12}\beta_{nn}
\end{pmatrix} = 0.
\]

Thus \( \alpha_{13}\alpha_{12}\beta_{34} = \alpha_{12}\alpha_{13}\beta_{34} = 0 \), since \( R[M] \) is reduced. Now multiplying \( \alpha_{12}\beta_{24} + \alpha_{13}\beta_{34} = 0 \) on the left by \( \alpha_{12} \), we obtain \( \alpha_{12}\beta_{24} = \alpha_{13}\beta_{34} = 0 \). Hence \( a_{rs}b_{st} = 0 \) for each \( r, s, t, i, j \geq 1 \), since \( R[M] \) is \( M \)-Armendariz. Thus \( a_{rs}cb_{st} = 0 \), for each \( c \in R \), since \( R \) is reduced. Consequently, \( A_{s}Cb_{j} = 0 \) for each \( C \in T_{4}(R) \). Therefore \( T_{4}(R) \) is \( M \)-quasi-Armendariz.

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