The Activities Based on van Hiele Model
Using Computer as a Tool

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The purpose of this article is to devise the activities based on van Hiele levels of geometric thought using computer software, Geometer’s Sketchpad (GSP) as a tool. The most challenging task facing teachers of geometry is the development of student facility for understanding geometric concepts and properties. The National Council of Teachers of Mathematics (Curriculum and Evaluation Standards for School Mathematics, 1991; Principles and Standards for School Mathematics 2000) and the National Research Council (Hill, Griffiths, Bucy, et al. 1989) have supported the development of exploring and conjecturing ability for helping students to have mathematical power. The examples of the activities built in GSP for students are designed to illustrate the ways in which van Hiele’s model can be implemented into classroom practice.

1. INTRODUCTION

The purpose of this article is to devise the activities based on van Hiele levels of geometric thought using computer software, Geometer’s Sketchpad (GSP) as a tool. The most challenging task facing teachers of geometry is the development of student facility for understanding geometric concepts and properties. The National Council of Teachers of Mathematics (1989; 1991; 2000) and the National Research Council (Hill, Griffiths, Bucy, et al. 1989) have supported the development of exploring and conjecturing ability for helping students to learn mathematical properties better. Examples of the activities built in GSP for students are designed to illustrate the ways in which van Hiele’s model can be implemented into classroom practice.

Traditionally, geometry in our textbooks has centered on fostering deductive reasoning abilities of students. The centerpiece of most geometry instruction is on geometric proof, which in many respects seems to be beyond the grasp of many students. Students copy
by memorizing theorems and proofs and come away from these experiences with no understanding and appreciation of either geometry or deductive reasoning and proof. Hoffer (1981) claimed that some geometry courses do not develop understanding but rather encourage memorization.

There is a strong hierarchy implied in the levels of geometric thought. One important aspect of van Hiele levels is that students at a lower level of thought can not be expected to understand instruction presented to them at a higher level of thought. According to van Hiele (1986), “this is the most important cause of bad results in the education of mathematics” (p. 66). If the van Hiele hypotheses about levels hold, then when students are asked to function at a higher level of mental development than they are capable of, they may adapt in several ways. Some may wish to please the teacher, just accept what the teacher says without any understanding, and resort to rote memorization. Others may reject the subject as something they cannot understand. Either way, poor attitudes and lack of understanding are the inevitable outcomes. Achievement in the latter case is usually poor and in the former it would depend on whether the teacher rewards rote learning or not. If this were the case, the achievement really has little value. There is clearly a need for geometry teachers to provide their students with the types of experiences necessary to enable students to make the transition to deductive geometry in a natural and meaningful way. Therefore, it seems very important for geometry teachers to know students’ levels of geometric thought and develop the activities, based on the van Hiele model.

Recent studies have begun to address the use of computer software as a tool in exploring concepts and properties of mathematics (Chazan 1989; Choi-Koh 1999; Jiang 1993). The pioneering study by Yerushalmy & Houde (1986) indicated that computer software facilitated the use of inductive reasoning by exploring and conjecturing properties and relations of geometry. Nowadays, inductive reasoning is extremely important in everyday life as well as in mathematics and science because it is the way we usually get ideas to try out and verify.

The availability of computers in mathematics provides a unique opportunity to develop useful methods for attacking problems with geometry. By exploring and conjecturing geometric ideas, students will become more engaged in subject matter and will become more skilled at inductive and deductive reasoning. With infusion of a tool such as the Geometer’s Sketchpad into geometry such an approach is feasible. Recently, some research (cf. Kang & Choi-Koh 1999) in Korea began to stress the availability of the software, GSP in math classrooms. Few studies however, have described instructional materials such as the activities in relation to van Hiele model and using the computer. Thus, this article was to develop the activities based on van Hiele model using dynamic computer software as a tool and intended to provide students with more experiences at
lower levels of geometric thought.

2. **Van Hiele’s Current Model**

Van Hiele has acknowledged that he is interested in the first three levels in particular rather than five levels of thought (Alan Hoffer, personal communication, February 25, 1985), and more recently van Hiele (1986) has described only three levels as existing in practicality for school mathematics. The naming of these three levels remains the same as in the earlier version: visual, descriptive and theoretical. It seems that the third level becomes extended to include the former last two levels.

Another important aspect of the van Hiele model, the five phases of the learning process, is detailed in the last article written by van Hiele-Geldof. Since the method and organization of instruction, as well as the content and materials used are important areas of pedagogical concern, instruction developed according to this sequence promotes the acquisition of a level (van Hiele-Geldof 1984).

In phase 1, information (inquiry), students and the teacher are engaged in conversation and activity about the objects of study. Observations are made, questions are raised and vocabulary is introduced.

In phase 2, directed orientation, the topic of study is explored through carefully sequenced materials and activities. Structures are gradually revealed. Tasks are short and responses are specific.

In phase 3, explication, students express in words the results of their work. The teacher’s role lies in “introducing the necessary technical terms” (van Hiele-Geldof 1984, p. 219). At this phase, accurate and appropriate language is encouraged.

![van Hiele’s current model of instruction](image-url)
In phase 4, free orientation, more complex, open-ended tasks with many steps and alternative solutions are included. Students use their creative abilities. They expand on what they have experienced. They order the things they must do to “obtain the intended result.”

In phase 5, integration, a review and summarization of what has been learned focuses on the new network of objects and relations.

The visual, descriptive and theoretical levels of thought and learning periods that lead to each of these levels are summarized in Figure 1 for this article. In the current model, once Teppo (1991) described, these levels are achieved by passing through different learning periods. During each period, students investigate appropriate objectives and engage in interactive learning activities designed to enable them to progress to the next higher level of thought.

3. CURRENT RESEARCH USING GEOMETRIC SOFTWARE, GSP IN KOREA

There has been much research using geometric software, Geometer’s Sketchpad in Korea. Kang & Choi-Koh (1999) devised the instructional materials that teachers can use for developing students’ spatial abilities. This enables students to build relationship of the figures between 2 dimensions and 3 dimensions by free movement of clicking action buttons in GSP. This is an example that GSP can also be used at the geometric figures of three dimensions with teachers’ understanding of caution in using it.

Bang (1997) included the problems using GSP to evaluate students’ problem solving ability and creative thinking in the study of the program development for selecting gifted students. According to Oh (1997), the geometric lessons using GSP were effective because the tasks drawing right pentagons and spiral enhanced students’ thinking levels. Joo (1997) demonstrated the ways of exploring geometric properties using GSP and then the deductive reasoning within teacher-centered instructions. In this case, she found that the students used GSP passively and just followed teacher’s instruction.

Jun & Joo (1998) suggested a geometric learning model, based on an exploratory learning using GSP applications. Such a model adopts GSP capability of visualizing dynamic geometric figures and the advantages of exploratory learning method in discovering properties and relations of geometric problem proving and concepts associated with students’ geometric inferencing. The research was done with three middle school students by applying the proposed model for six times of the period at a computer laboratory. The research indicated that the students with less than van Hiele 4 level took advantages of adopting the proposed model to gain concrete understandings of geometric principles and concepts using GSP.
4. Results

4.1. Level 1 – Visual

Students recognize shapes globally. It is possible to see isosceles triangles, but it is senseless to ask why they are isosceles. There are no ways, but one just sees it.

Learning Period 1

Students move from level 1 to level 2 thought. The objectives of study during this period consist of properties of individual figures. For example, students begin to recognize that an equilateral triangle contains three congruent sides and has three perpendicular bisectors that are lines of symmetry to each side.

First phase: Information

Activity: Figure 2

Materials related to the level 1 of study are presented to the students as prerequisite knowledge to Period 1. Students functioning at this level can learn geometric vocabulary, can identify specified shapes, and given a figure, can reproduce it. For example, given the pictures in Figure 2, a student would be able to recognize that there are isosceles triangles in (a), equilateral triangles in (b), and right triangles in (c).

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2}
\caption{Giving each group a name}
\end{figure}

Second phase: Directed orientation

Activities: Figures 3 & 4
1. The students start to explore the field of inquiry through carefully constructed figures. Find the line of symmetry in each figure in Figure 3.

Students can try to associate a line of symmetry with various shapes of triangles. They demonstrate the reflection of a shape about line $m$ using GSP and show how this
reflection affects given an object with the correspondent one. They discuss properties of shapes revealed by reflection.

Figure 3. Symmetric figures

2. With carefully constructed three kinds of triangles, find the properties of each triangle.
   What properties must triangle A have so as to exhibit the line of symmetry?
   What properties must triangle B have so as to exhibit the line of symmetry?
   What properties must triangle C have so as to exhibit the line of symmetry?

Figure 4. Isosceles, equilateral and right triangles

Third phase: Explication

Students and the teacher engage in discussion about properties of three triangles. We call triangle A the isosceles triangle, triangle B the equilateral triangle and triangle C the
right triangle.

**Fourth phase: Free orientation**

Activity: Figure 5

The students are given two vertices of a triangle as more open-ended activity that can be approached by several different types of solutions. Find the third vertex to have

a) an isosceles triangle
b) an equilateral triangle, and
c) a right triangle.

![Figure 5](image)

*Figure 5. Given two vertices, find the third*

**Fifth phase: Integration**

The teacher helps the student to gain an overview of the field of study and to integrate the subject matter investigated. Students summarize all properties they work on and are able to discern a triangle by its own properties.

**4.2. Level 2 – Descriptive**

Students distinguish shapes on the basis of their properties.

**Learning Period 2**

Students move from level 2 to level 3 thought during this period. The objectives of study are networks of relationships and the ordering of properties of geometric figures. Using informal deductive reasoning, students become able to prove relationships.

**First phase: Information**

Activity: Figure 6
Students use the line of symmetry of an isosceles (or equilateral) triangle to construct such a figure when the base of a triangle is given (see Figure 6).

![Figure 6. Completing a triangle](image)

**Second phase: Directed orientation**

Activities: Figures 7 & 8

1. Shapes of triangles (Figure 7)
   a) Find the class inclusion among triangles. Can isosceles triangles be called equilateral triangles? Or can equilateral triangles be called isosceles triangles?
      What about right triangles related to other triangles?
   b) Express the definition for all triangles including acute, obtuse, and scalene triangles.

![Figure 7. Shapes of triangles](image)

2. Area of a triangle
   Students understand the area of a triangle is the half of a quadrilateral (refer to Figure 8).
Third phase: Explication

Students and the teacher engage in discussion about relations among three triangles. All equilateral triangles can be isosceles triangles, but isosceles triangles can’t be equilateral triangles. A right triangle can be isosceles in case two sides are equal. Also, the students explain that the area of a triangle is always $1/2 \times \text{width} \times \text{length}$.

Fourth phase: Free orientation

Activity: Figure 9

Figure 8. Area of a triangle

Figure 9. Constant area of triangles
The students are given more open-ended activity that can be approached by several different types of solutions because of availability of multiple representations. That is, anytime students move one vertex, they face a new situation in which a different triangle $ABC$ is shown, but always constant areas are found with animation button activated. Students discover informal deduction that are needed to determine that the area of a triangle is constant even though the shape of the triangle has kept changing as top vertex $A$ moves along the segment $DE$ parallel to the base of $\triangle ABC$.

**Fifth phase: Integration**

As completing instruction for the period 2, the teacher helps the students to gain an overview of the field of study and to integrate the subject matter investigated. Students summarize all relations they work on: They are able to discern triangles by definitions, implications, and class inclusions, to formulate the area of a triangle, and to reason out to the constant area that the areas of triangles having the same altitude and the same base are equal.

### 4.3. Level 3 – Theoretical

Students are able to devise a formal geometric proof and to understand the process employed (van Hiele 1986, p. 86). For this level, the following activities can be introduced during the instruction for developing formal deductive reasoning and for applying what they understand about the centroid to more open-ended problems.

**Activities: Figures 10 & 11**

*Figure 10. Six triangles by the centroid*
5. Conclusion

5.1. Two Learning Periods

The intent of the period 1 was to help students develop geometric thought resulting in the transition from level 1 to level 2. The students will progress toward level 2 and be able to analyze figures in terms of their properties and relations among components and to discover properties/relations of class of shapes empirically by measuring and observing dynamically transforming figures.

The period begins with symmetry as intuitive instruction. In much of the early part of period 1, dynamic, multiple representations of GSP help the students to develop understanding out of their perception in discovering the properties of each triangle by facilitating students’ verbalizing the properties, based on lines of symmetry, and to easily correct mistakes by providing counter-examples. The symmetry property as a basic concept for the study is devised to help the students understand the properties of each triangle as well as the properties of congruence by reflected figures in GSP.

Through active discussion with the researcher, the students will engage in development of geometric thought. Dynamic, multiple representations in GSP will provide the students the opportunity to discuss their orientation and to find their way in the field of thought. As the period goes on, the construction of each triangle, based on the properties they discover previously, encourage the students to develop their understanding of geo-
metric figures. If students attempted to learn which properties belong to given figures and how to act in a given situation, the students will have a network of relations at their disposal that is sufficiently connected to the original field of thought. Such students, starting from given concrete situation, will not have any difficulty returning to the corresponding significance.

Constructing figures in the period 1 must be effective and desirable for the students in developing the van Hiele level 2 of geometric thought as well as logical geometric thought. In this period, however, the possibility of logical thought must not be overestimated. It is impossible that at this level the students can survey the whole of the reasoning because the relations between the figures have not yet obtained understanding. The software helps the students to develop understanding out of their misunderstanding during constructing figures with active discussion of the properties of a figure.

The intent of the period 2 is to help students develop geometric thought making the transition from level 2 to level 3. Notably, constructing a centroid in a triangle and discovering the properties of the center can be very productively done in GSP. Without the software as a tool, it will not be easy for the students to explore the properties of the center, because of the limits of construction by paper and pencil. For example, the students, using GSP, draw a median in a triangle from the vertex angle to its opposite side and conjecture that the median bisects the area of a triangle, based on the activity of constant area of triangles in period 2. Then, the student can measure the areas in question and test the conjecture further by drawing other medians on the other sides of the triangle. Data obtained in this way form the basis for a conjecture that the student believes is true. Through this inductive reasoning process, they can demonstrate development of a logical, deductive system of geometric thought; six triangles drawn by three medians have a congruent area and at last, the centroid divides the medians into ratio 2:1 from each vertex.

Also, the open-ended problems are planned to enhance the students’ ability to apply and generalize their conjectures to various situations and to observe their reasoning process in the phase of free orientation. The students may enjoy developing reasoning for formal deduction based on previously discovered properties of the center. In other words, when they develop hierarchical van Hiele levels of geometric thought, they may reach the formal deductive systems in all triangles they learn.

5.2. The Role of Inductive Reasoning

The students will be given an opportunity to explore and conjecture geometric properties while they work on the activities prepared in this article to develop their understanding throughout the two periods of learning. What students behave as active learners is important because working on the activities offers a wealth of visual and
numerical data and conjectures about properties and relations of figures can be tested quickly within data.

Constructing figures is the most crucial and effective in the period 1, since it helps students to focus intensively on specific components and details of complex problems. Students can easily develop understanding by not only observing, discussing, interpreting visual and numerical data, but also by conjecturing figures. Those inductive reasoning processes play an important role in students’ abstracting of understanding properties of each triangle in a next higher level of thought. Also, students can begin to see the possibilities of logical thought grounded by deductive reasoning of relations in figures. In the context of the constant area, if inconsistent changes of values occur, then students will have an opportunity to compare the value measured as the result of their formula with the value originally given by the function in GSP and to keep generating reasoning until they match.

5.3. Implications for Instruction

The two learning periods in this study, based on van Hiele’s phases of learning that lead a student from one thought level to the next, are prototype materials and give examples of the steps in the learning cycle. Quite clearly, the ability to think at higher levels is not acquired from written materials alone. The phases of learning instead suggest an interaction between students and a teacher similar to one in which a doctor provides his or her patient with adequate prescription.

Technology in mathematics classrooms must be an effective tool to enhance this interaction between students and the teacher by allowing them to investigate their conjectures. Teachers can motivate and enrich students’ learning by flexible use of the five phases of learning in regular tasks and open-ended problems. In order to reach their goal, obviously teachers should be knowledgeable about their students’ learning levels and the content areas they are teaching so that they can effectively use this flexibility.

As a method of this instruction (how), the van Hiele phases of learning seem to provide a complete teaching and learning plan when a dynamic tool (software) is used. For example, conjecturing environment with visual representations carefully sequenced can allow students to discover properties of a figure: the phase 2, directed orientation. Students dynamically test their conjectures and are encouraged to express their findings, while a teacher introduces new terminology: the phase 3, explication. Problem-solving activities are prominent where teachers expect students to find their own ways through the function of immediate self-feedback in the software: the phase 4, free orientation. Reviewing and synthesizing what they have learned take place at ease: the phase 5, integration. The phases are not usually accomplished in a linear fashion; rather, students
frequently cycle over several phases more than once before attaining a new domain of thought, and finally reach the next level.

Since the focus of this article is on providing sufficient opportunities in the two periods given between the first three levels of geometric thought, the activities devised in the article must be useful instructional materials in mathematics lessons. The Seventh Mathematics Curriculum Reform, which is implemented from the year 2001 into the secondary schools, suggested the active use of technology in math classrooms. As we know, our textbooks in geometry portion more time on the formal deduction and do not provide students the opportunities to use this kind of dynamic tool. Before they work on formal deductive reasoning, the students should be exposed to these kinds of activities more often. Also, teachers may get ideas out of this article in order to create some sub-periods after the third level of geometric thought, in which their students hierarchically progress.

REFERENCES

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