The 1st International Workshop on Mathematical Sciences: Mathematical Analysis and its Applications

January 6-7, 2006, Sogang University, KOREA

Program

	6	7
9:10∼9:30	Opening	
9:30∼10:10	M. Kunzinger	A. Tamburrino
10:20∼11:00	R. Steinbauer	T. Takiguchi
11:10∼11:50	J. Byeon	O.-I. Kwon
12:00∼12:40	H. Kim	K. Yoshino
	Lunch	
2:10∼2:50	W.S. Kim	J. Chung
3:00∼3:40	H.-O. Bae	C.E. Shin
	Coffee Break	Coffee Break
4:00∼4:40	S. Cho	N.K. Lee
4:50∼5:30	J.S. Lee	Y.-S. Chung
6:00∼		Banquet

Michael Kunzinger | Department of Mathematics, University of Vienna

Nonlinear distributional geometry

Applications of the theory of distributions to global analysis and differential geometry have a long history, starting with the work of Schwartz and de Rham and continued by Lichnerowicz, Marsden, Parker, and others. Due to the nonlinear nature of the basic operations of tensor analysis lying at the core of many geometric applications, a purely linear distributional geometry encounters a number of fundamental limitations.

Algebras of generalized functions offer a way of overcoming these obstacles and have over the past few years supplied the foundations for the development of a nonlinear distributional geometry adapted to the needs of nonsmooth differential geometry and its applications in mathematical physics, in particular in general relativity (cf., e.g., [1,2,3,4]).

In this talk we provide a survey of these recent developments and give an outlook on current directions of research and open problems in the field.

References

[1] M. Grosser, M. Kunzinger, M. Oberguggenberger, R. Steinbauer, *Geometric Theory of Generalized Functions with Applications to General Relativity,* Mathematics and its Applications 537, Kluwer, (2001).

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Roland Steinbauer | Department of Mathematics, University of Vienna

Weak solution concepts for some kinetic equations

In kinetic theory one often considers large ensembles of collisionless particles which interact only by fields they create collectively. This situation commonly is referred to as the mean field limit of a many-particle system and is mathematically modelled by the Vlasov equation coupled to the respective field equations. There is an extensive literature on the resulting systems of PDEs; cf. e.g. [1].

In this talk we first discuss some recent results on the Vlasov-Klein Gordon system. In particular, we present a local existence result for classical solutions ([3]) as well as a global result for weak solutions with initial data statisfying a size restriction ([2]).

Then we turn to existence of generalized solutions (in the sense of J.F.Colombeau) of the Vlasov-Poisson system ([4]) and discuss possible applications to the singular limits of this system which are the Euler-Poisson system with vanishing pressure ([6]) as well as the n-body problem ([5]).

References

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Jaeyoung Byeon | Department of Mathematics, POSTECH

Spike layer solutions for singularly perturbed nonlinear elliptic problems

For a domain $\Omega \subset \mathbb{R}^N$, we consider the following equation

$$\varepsilon^2 \triangle v - V(x)v + f(v) = 0, \quad x \in \Omega \tag{1}$$

with some boundary conditions on $\partial\Omega$. When ε is sufficiently small, for certain typical nonlinearity f there are many kinds of solutions due to the geometry of Ω or potential V. In this talk, I would like to introduce my recent results on the construction of certain solutions for almost optimal nonlinearity f when ε is sufficiently small.

Hyunseok Kim | School of mathematics, Korea Institute for Advanced Study, Korea

On stability of the plane Couette flow

The stability of the plane Couette flow has been an important subject in the theory of hydrodynamical stability. This stability problem is closed related, via Squire transform, to the eigenvalue problem of the Orr-Sommerfeld equation. In a classical paper by Romanov (1973), it was shown that all eigenvalues of the Orr-Sommerfeld equation. In a classical paper by Romanov (1973), it was shown that all eigenvalues of the Orr-Sommerfeld equation should lie in the open left half-plane. From this important result, Romanov could conclude that the plane Couette flow is stable with respect to perturbations small in the H^1-norm. On the other hand, the analyticity of the Stokes semi-group in $L^q_\sigma(\mathbb{R}^2 \times (-1,1)), 1 < q < \infty$, has been established by Abe and Shibata (2003) and Abels and Wiegner (1994, 2004), independently. Hence it may be *conjectured* that the plane Couette flow is stable with respect to perturbations small in the L^3-norm.

The purpose of this talk is to give a partial answer to the conjecture. The stability is shown in case when the perturbations are small in the L^3-norm and periodic in $x' = (x_1, x_2)$. The key observation is that in the periodic setting, the perturbed Stokes operator has compact resolvents and so its spectrum consists entirely of isolated eigenvalues. This observation, combined with Squire transform and Romanov's result, allows us to deduce that the spectrum is independent of q and lies in the open left half-plane. Then our L^3-stability result may be proved by the standard iteration method due to Kato.

Wan Se Kim | Hanyang University

Multiple existence of solutions for semilinear elliptic problem

In this paper, we discuss three important techniques for the solvability of some semilinear differential equations which are subject to prescribed boundary conditions. We will exploit three essentially distinct techniques, variational method, degree method and super- subsolution method.

Hyeong-Ohk Bae | Ajou University

Pressure Representation and Boundary Regularity of the Navier-Stokes Equations with Slip Boundary Conditon

We first represent the pressure in terms of the velocity in $\mathbb{R}^3_+$. Using this representation we prove that a solution to the Navier-Stokes equations is in $L^\infty(\mathbb{R}^3_+ \times (0,\infty))$ under the critical assumption that $u \in L^{r,r'}_{loc}$, $\frac{3}{r} + \frac{2}{r'} \leq 1$ with $r \geq 3$. Choe 1998 showed that a boundary L^∞ estimate for the solution is derived if the pressure on the boundary is bounded. In our work, we remove the boundedness assumption of the pressure. Here, our estimate is local. Indeed, employing Moser type iteration and the reverse Hölder inequality, we find an integral estimate for L^∞-norm of $\mathbf{u}$.

Sanhyun Cho | Department of Mathematics, Sogang University

Extension of CR structures on three dimensional compact pseudoconvex CR manifolds

Let M be a smooth compact orientable pseudoconvex CR manifold of real dimension three and assume that there is a smooth function λ which is strictly subharmonic in any direction where the Levi-form vanishes on M. Then we extend the given CR structure on M to an integrable almost complex structure on the concave side of M. As an application, if M is a non-compact pseudoconvex CR manifold of real dimension three, we prove that the given CR structure on M can be locally extended to an integrable almost complex structure on the concave side of M.

Jaesung Lee | Department of Mathematics, Sogang University

Bochner-Martinelli integral and Layer Potential in the complex unit ball

We give an elementary proof of the statement that a function f on the closed unit disc of $\mathbb{C}^n$, integrable on the unit sphere, is holomorphic if it is invariant under the Bochner Martinelli integral transform and we prove that the holomorphicity of a single layer potential function Tf is equivalent to the holomorphicity of its moment function f.

Antonello Tamburrino | Università di Cassino

A fast method for solving the inverse medium problem in electrical resistance tomography and in magnetic induction tomography

Electrical resistance tomography (ERT) and magnetic induction tomography (MIT) (see figure 1) are two techniques capable of providing the spatial distribution of the electrical conductivity ρ and of the magnetic permeability of the interior of a material body, starting from measurements taken externally to the body itself (nondestructive evalution, NDE). The field of NDE is receiving increasing attention since the last decades; the list of applications field where is used is continuously increasing. From the mathematical viewpoint, the reconstruction of the material properties of a body starting from external measurements is a nonlinear and inverse problem. The methods for processing the measurements to obtain the spatial distribution of the sought material property must "fight" against the ill-posedness of the problem and, in many situations, they are not adequate for real-time processing.

The contribution of our researches in developing fast inversion method is based on a monotonicity property of the operator mapping, for instance, the resistivity into the measured quantity [1-4]. For instance, in the case of ERT, assuming as data the matrix of the mutual resistances between pairs of electrodes, we have [1, 2]:

$$\rho_1(\mathbf{r}) \leq \rho_2(\mathbf{r}) \text{ in } \Omega \Rightarrow \mathbf{R}_1 \leq \mathbf{R}_2 \qquad (1)$$

where $\Omega \subseteq \mathbb{R}^3$ is the domain occupied by the material body and $\mathbf{R}_k$ is the resistance matrix when the resistivity is ρ_k. Monotonicity (1) follows from the elliptic nature of the governing equation for ERT. Monotonicity (1) for the Dirichlet-to-Neumann map is known from long time (see [3], for instance). MIT is governed by a parabolic PDE. Nevertheless, in case the unknown quantity is still the resistivity of the body, (1) is still valid (see [4]-[6]) after the subsitution $\mathbf{R}_1 = \mathbf{P}_2^{(2)}$ and $\mathbf{R}_2 = \mathbf{P}_1^{(2)}$ where $\mathbf{P}_k^{(2)}$ is the second order term of the series expansion w.r.t. the angular frequency ω, of the mutual impedance matrix $\mathbf{Z}_k$ obtained when the resistivity is ρ_k:

$$\mathbf{Z}_k(\omega) = \mathbf{R}_0 + j\omega\mathbf{L}_0 + \omega^2\mathbf{P}_2^{(2)} + O(\omega^3) \text{ for } \omega \to 0. \qquad (2)$$

Property (1) has been the basis for developing a fast inversion method for two-phases materials. This method, in its first version, requires the solution of a number of forward problems proportional to the number of voxels used to discretize the unknown [1], [2]-[6]. Then, thanks to a minor modification of the methods, we have obtained a new version that is appeared suitable for real-time imaging since, during the computation, it does not requires the solution of forward problems [4] but only precomputed matrices. The number of precomputed matrices is equal to the number of voxels representing the unknown.

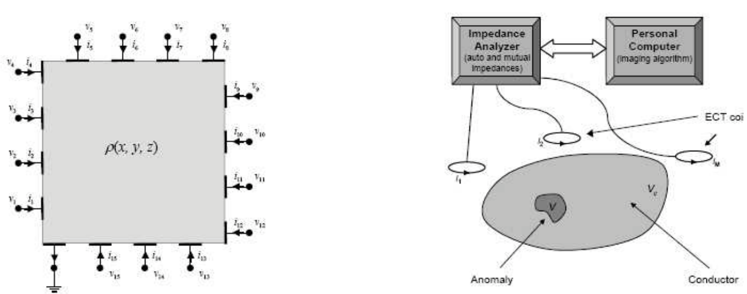

Fig. 1. Left: ERT configuration. Measurements of the self and mutual resistances between electrodes placed in contact with the boundary are collected. Right: MIT configuration. Time-harmonic measurements of self and mutual impedances between coils are collected at several frequencies.

Takashi Takiguchi | Department of Mathematics, National Defense Academy of Japan

Some remarks on Lorentz x-ray problem in the frame of the rotated axes

In this talk, we discuss reconstruction of measurable plane sets from their two projections. Let $F \subset \mathbb{R}^2$ be a measurable plane set such that $\lambda_2(F) < \infty$, where λ_i is the Lebesque measure on $\mathbb{R}^i$. Let $f(x, y)$ be the characteristic function of F. Define the *horizontal* and the *vertical projections* of F (or equivalently of f) as

$$PF(y) := \int_{-\infty}^{\infty} f(x, y)dx \tag{1}$$

and

$$QF(x) := \int_{-\infty}^{\infty} f(x, y)dy, \tag{2}$$

respectively. *The reconstruction problem* of measurable plane sets from their orthogonal projections is as follows:

Problem 1. *Given two non-negative, integrable functions f_1 and f_2 having the same L^1 norm, find a measurable plane set f such that $PF = f_1$ and $QF = f_2$ almost everywhere.*

This problem was first studied by G.G. Lorentz in 1949, in view of which, let us call this problem *Lorentz x-ray problem*.

As an example of the application of this problem, consider a homogeneous object in the three dimensional space which contains a hole in its interior. We would like to detect the hole without destracting the object itself. Study this problem on the section by a plane and apply the x-ray tomography for this reconstruction. Then the results in this problem are applied to solve this problem.

There are a number of studies on this problem. We first introduce these known results. If we study Problem 1 only in the frame of the orthogonal axes, the sets we can treat is very limited. Therefore there arises the necessity to study this problem in the frame of the axes which are not necessary orthogonal. We call this frame *the frame of the rotated axes*. In this talk, we mention this necessity and extend the known results on Problem 1 in the frame of the rotated axes. We also mention some remarks on the original results.

It seemed to the author that any set can be reconstructed in the frame of the rotated axes, if the rotation angles are suitably chosen, however, it turns out that it is false. We also mention this problem.

Oh-In Kwon | Department of Mathematics, Konkuk University

Conductivity and current density imaging in magnetic resonance electrical impedance tomography

This talk introduces the latest impedance imaging technique called Magnetic Resonance Electrical Impedance Tomography (MREIT) providing information on electrical conductivity and current density distributions inside and electrically conducting domain. We briefly summarize the related technique of Electrical Impedance Tomography (EIT) that deals with cross-sectional image reconstructions of conductivity distributions from boundary measurements of current-voltage data. Noting that EIT suffers from the ill-posed nature of the corresponding inverse problem, we introduce MREIT as a new conductivity imaging modality providing images with better spatial resolution and accuracy. MREIT utilizes internal information on the induced magnetic field in addition to the boundary current-voltage measurements to produce three-dimensional images of conductivity and current density distributions. We will introduce a new method to measure magnetic flux density B_z in MREIT. The proposed method doesn't use phase distortion effect of B_z on ordinary NMR signal, but utilize frequency distortion from B_z that comes from injected current. We will analyze the distortion and develop an efficient algorithm to get back B_z.

Kunio Yoshino | Department of Mathematics, Sophia University

Recent development of heat kernel method in the theory of generalized functions

In 1987, Professor Tadato Matsuzawa introduced the heat kernel method for generalized functions ([1], [2], [3]).

He succeeded to simplify the theory of hyperfunctions.

The group conducted by D. Kim, S. -Y. Chung, and J. Chung developed Matsuzawa's idea. They obtained several results by using heat kernel method.

During my talk I will treat the following things.

1. *Characterization of generalized functions by heat kernel method*
2. *Paley-Wiener type theorem for generalized functions*
3. *Bochner-Schwartz theorem for positive definite generalized functions*
4. *Micor local analyticity of positive definite distributions*

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Jaeyoung Chung | Department of Mathematics, Kunsan National University

On a class of Pexiderized functional equations in distributions

In this talk we first consider a distributional analogue of an n-dimensional version (without loss of generality the following simplified form) of the functional equation of Aczél and Chung

$$\sum_{j=1}^{l} f_j(x+\beta_j y) = \sum_{k=1}^{m} g_k(x)h_k(y) \tag{1}$$

where $f_j,\, g_k,\, h_k : \mathbb{R}^n \to \mathbb{C}$ and $\beta_j \in \mathbb{R}^n$ for $j = 1, \ldots, l$, $k = 1, \ldots, m$. For $\beta_j = (\beta_{j1}, \ldots, \beta_{jn})$, $y = (y_1, \ldots, y_n) \in \mathbb{R}^n$ we denote by $\beta_j\, y = (\beta_{j1}y_1, \ldots, \beta_{jn}y_n)$, $j = 1, \ldots, l$.

Secondly we consider a distributional version of the following Pexiderized logarithmic functional equations

$$f(x+y) - g(xy) = h(1/x + 1/y), \quad x, y > 0, \tag{2}$$

$$f(x+y) - g(x) - h(y) = k(1/x + 1/y), \quad x, y > 0, \tag{3}$$

$$f\left(\frac{x+y}{2}\right) + g\left(\frac{2xy}{x+y}\right) = h(x) + k(y), \quad x, y \in I, \tag{4}$$

where $I \subset (0, \infty)$ is an open interval.

Thirdly we consider a distributional version of the stability of the most general form of quadratic functional equation

$$\|f(x+y) + g(x-y) - 2h(x) - 2k(y)\|_{L^\infty} \le \epsilon. \tag{5}$$

Chang Eon Shin | Department of Mathematics, Sogang University

Method of nonuniform sampling theorem

We consider entire functions f whose Fourier transforms have compact supports. We introduce the method to derive sampling expansions of f, that is, by means of, orthonormal basis, Riesz basis and contour integral.

Nam Kee Lee | Seoul National University

Inverse source problems on networks

A network represents a way of interconnecting any pair of users or nodes by means of some meaningful links. Thus, it is quite natural that its structure can be represented, at least in a simplified form, by a connected graph whose vertices represent nodes and whose edges represent their links.

For example, the brain is a network of neurons; organizations are people networks; the global economy, food webs, molecules, and the internet can all be represented as networks.

In this talk, we will introduce an elliptic operator on the graph, the ω - Laplacian Δ_ω and interpret it as a diffusion equation on the graph modeled by the electric network. Using tools of partial differential equations, we have some results on direct problems about Laplacian equation, Poisson equation and heat(diffusion) equation.

And we deal with inverse source problems about Poisson equation and heat(diffusion) equation on network to identify the electric source or the heat(diffusion) source of network by using Neumann data.

References

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Yun-Sung Chung | Dept. of Mathematics and Program of Integrated Biotechnology, Sogang University

On an inverse source problem for the heat equation

Let $\Omega \in \mathbb{R}^n$, $n = 2$ or 3 be a bounded domain. The main purpose of this talk concerns the problem of identifying the source term F in the following parabolic problem

$$(\partial_t - \Delta)u(x,t) = F(x,t) \quad \text{in } \Omega \times (0,T)$$

$$u(x,0) = 0 \quad \text{in } \Omega$$

$$u(x,t) = 0 \quad \text{on } \partial\Omega \times (0,T)$$

$$\frac{\partial u}{\partial n} = \phi \quad \text{on } \partial\Omega \times (0,T).$$

In this talk, the source term F will be considered as

$$F(x,t) = \sum_{l=1}^{n} \sum_{k=1}^{m_l} \lambda_{l,k}(t)\ \delta^{(l)}(x - S_{l,k})$$

where δ is a Dirac distribution. Here, the locations $S_{l,k}$ and the intensities $\lambda_{l,k}(t)$ of the source are unknown. By applying an algebraic method, we first compute locations $S_{l,k}$ and then determine the intensities $\lambda_{l,k}(t)$ by their exponential moments.